You should work on this lab in groups of three or four students and turn it in at the beginning of lab next week, September 27.

There is a test on Friday of this week. The questions in the first part of the lab are relevant to the test, and you should complete them before the test. The second part of the lab covers new material on which you will not be tested.

Part 1: Three omnibus review problems.

1. Determine the behavior of the following rational function at ∞ and find its horizontal asymptote, if any. Determine the behavior of the function at points where the denominator is zero; at each point find the limit (or the limits from the left and right) and determine whether there is a vertical asymptote at that point.

$$g(x) = \frac{(x^2 - x - 2)(6x^4 + 3x^2 + 7)}{(x-3)(3x^3 - 12x)(x^2 - 2x + 3)}$$

2. Consider the following table that shows some values of a function $f(x)$. Make plausible guesses for whatever limits you can, based on the information in the table (including one-sided limits, infinite limits, and limits at infinity). *Explain your answers.*

x	$f(x)$
-1000	3200
-100	125
-10	9
-1.1	3.207
-1.01	3.065
-1.001	3.013
-1.0001	3.002
-1	2

x	$f(x)$
0.75	2.470
0.9	2.284
0.99	2.2512
0.999	2.2501
1	2.25
1.001	2.2495
1.01	2.238
1.1	2.225

x	$f(x)$
2.1	-1.7
2.01	-16.25
2.001	-1581
10	7
100	7.1
1000	6.94
10000	7.002
100000	6.999

3. Draw a graph of a function $y = h(x)$ that satisfies all of the following:

a) For all values of a not mentioned, $h(a)$ and $\lim_{x\to a} h(x)$ are both defined and are equal

b) $\lim_{x\to\infty} h(x) = -\infty$ **c)** $h(1) = 4$ **d)** $\lim_{x\to 1^-} h(x) = 2$ **e)** $\lim_{x\to 1^+} h(x) = 2$

f) $\lim_{x\to -\infty} h(x) = 1$ **g)** $\lim_{x\to 3^-} h(x) = \infty$ **h)** $\lim_{x\to 3^+} h(x) = -\infty$ **i)** $\lim_{x\to -2} h(x) = \infty$

j) $f(0) = 2$ **k)** $\lim_{x\to 0^-} h(x) = 2$ **l)** $\lim_{x\to 0^+} h(x) = -1$ **m)** $f(-4) = 2$

Part 2: Limits, sequences, and approximation.

*We have been looking at limits of functions $f(x)$ as $x \to \infty$, but it's also possible to look at functions $f(n)$ that are defined only for positive integers n and see what happens as $n \to \infty$. Such functions are called **sequences**. We won't study sequences formally, but you can investigate what happens in some cases...*

4. Suppose that whenever a certain ball falls from a height h, it will bounce back to height $\frac{1}{2}h$. It will then fall and bounce back to height $\frac{1}{4}h$, and so on. Each bounce is one-half the height of the previous bounce. Theoretically, it bounces an infinite number of times. Suppose that you drop the ball from a height of 2 feet. How far will it travel in toto, in an infinite number of bounces? To investigate this question, compute how far it bounces after one bounce, two bounces, three bounces, etc., and guess what what happens as the number of bounces approaches infinity. Since it bounces an infinite number of times, do you think it will keep bouncing forever? Explain.

5. Complete "Guided Project 9," which you can find on the back of this page. This project is one of a series of suggested in the supplementary material for our textbook.