This lab is to be turned in at next week's lab period, on November 2. Present your responses fully and clearly.

1. The book uses implicit differentiation to prove the derivative formula $\frac{d}{dx}\,x^r = rx^{r-1}$ for rational numbers r, but it might be a little confusing. It's a little less confusing to look at $\frac{d}{dx}\,x^{1/n}$ for a positive integer n. Suppose $y = x^{1/n}$ where n is a positive integer. Prove that $y' = \frac{1}{n}x^{(1/n)-1}$ as follows: Write $y^n = x$. Apply implicit differentiation to this equation, solve for y', and show how to rewrite the result in the desired form.

2. We have a new derivative rule: If b is a positive constant, then $\frac{d}{dx}\,b^x = b^x \ln(b)$. Using this rule and other rules, compute each of the following derivatives:

a) $\dfrac{d}{dx}\cos\left(2^x - 3^x\right)$ **b)** $\dfrac{d}{dx}\,2^{2^x}$ **c)** $\dfrac{d}{dt}\,t^5 5^t$

3. Recall that *even* functions such as $f(x) = x^2$ and $f(x) = \cos(x)$ satisfy $f(-x) = f(x)$, and *odd* functions such as $f(x) = x^3$ and $f(x) = \sin(x)$ satisfy $f(-x) = -f(x)$. What happens when you take the derivative of an even or odd function? Try some examples. Invent a theorem about the relationship between odd/even and derivatives. Then **prove** your theorem.

4. Suppose that $f(x)$ is a function, and $g(x)$ is defined as $g(x) = f(|x|)$.

a) How does the graph of $g(x)$ compare to the graph of $f(x)$? Give some specific examples. Come up with a general description, and explain why your description is valid.

b) Can $g(x)$ be differentiable at $x = 0$? (The answer is yes.) Try some examples, including $f(x) = x$, $f(x) = x^2$, and $f(x) = x^3$. Come up with a theorem that gives a condition under which $g'(0)$ will exist. Then **prove** your theorem.

5. This problem concerns tangent lines through points that do **not** necessarily lie on a curve. Consider the curve $y = x^2$.

a) The tangent line to $y = x^2$ at $x = a$ has slope $2a$ and passes through the point (a, a^2). Find the equation for this tangent line, in the form $y = mx + b$, where m and b are expressed in terms of a.

b) Verify that the tangent lines to $y = x^2$ at $a = 1$ and at $a = 5$ both pass through the point $(3, 5)$.

c) Now, suppose that you are given a point (c, d), and you want to find all lines that pass through (c, d) and are tangent to $y = x^2$. Use the equation for tangent lines that you found in part **a**. Plug c for x and d for y into that equation, and solve for a in terms of c and d. (Use the quadratic formula.) Explain the meaning of your answer in terms of tangent lines.

d) Use the result from part **c** to find all tangent lines to $y = x^2$ that pass through the point $(1, -3)$.

e) What happens if you try to use the result from part **c** to find tangent lines to $y = x^2$ that pass through the point $(1, 3)$?

f) How many tangent lines to $y = x^2$ pass through the point (c, d)? The answer can be zero, one, or two. For which points (c, d) is the answer zero? one? two? Explain why the answer makes sense geometrically. Draw a picture.

g) For the curve $y = x^3 - 3x$, three different tangent lines can intersect at a point. Draw a picture to illustrate this (without calculating or proving your result).