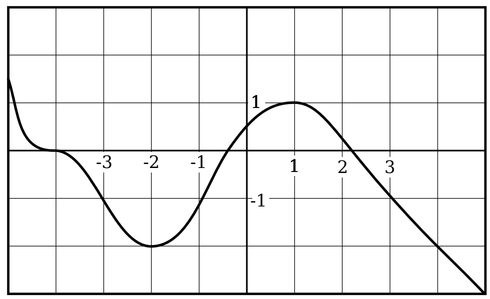
This lab is meant as review for tomorrow's test. It will not be collected. Sample answers will be available at the lab. Note that this is meant to be a general review of the material you should know; however, not every type of problem from this lab will appear on the test and not every problem on the test will be similar to a problem on this lab. There are more questions here than would fit on a test!

1. Consider the function $f(x)$ whose graph is shown here:



a) Find all values of x for which $f'(x)$ is zero.

b) For which values of x is $f'(x) > 0$?

c) Which is larger, $f'(-1)$ or $f'(0)$?

d) If you had to take a guess, what would you say is $\lim_{x\to+\infty} f'(x)$?

2. Let $f(x) = \begin{cases} 3x^2, & \text{if } x < 1 \\ x^3 + 2, & \text{if } x \geq 1 \end{cases}$. Show $f(x)$ is continuous at $x = 1$ but not differentiable at $x = 1$.

3. A point moves along a line so that its position at time t is given by $s(t) = 10(1 - e^{-t/5})$. Find formulas for the velocity, $v(t)$, and the acceleration, $a(t)$, at time t. Does the point move towards the left or towards the right? Why?

4. Let a be the constant $e^{1/e}$. That is, a is e raised to the power $\frac{1}{e}$ Show that the graph of $y = a^x$ contains the point (e, e) and that the tangent line to the graph of $y = a^x$ at (e, e) is the line $y = x$.

5. Compute $\dfrac{d}{dx}\, e^{2x}$, $\dfrac{d^2}{dx^2}\, e^{2x}$, and $\dfrac{d^3}{dx^3}\, e^{2x}$. What is a general formula for $\dfrac{d^n}{dx^n}\, e^{2x}$?

6. Suppose that $F(x)$ is a function that satisfies $F'(x) = e^{-x^2}$.

a) Compute $F''(x)$.

b) Define $r(x) = F(3x^2 + 1)$. Compute $r'(x)$, and find the value of $r'(1)$.

7. What is meant by an *inverse function*. Give a specific example.

8. Discuss what it means to say that a derivative represents a *rate of change.*

9. Compute the following derivatives. They can all be done using rules that you should have memorized.

a) $\frac{d}{dx}\left(2x^5 - 3x^7 + 4\right)$ **b)** $\frac{d}{dx}\left(3x^{7/3} + 7x^{3/7}\right)$ **c)** $\frac{d}{dt}\left(\frac{\sin(2t) - \cos(3t)}{t + e^t}\right)$

d) $\frac{d}{dx}\, e^{2x}\cos(e^x)$ **e)** $\frac{d}{d\theta}\left(\theta^3 + \theta\sin(\theta)\right)$ **f)** $\frac{d}{dy}\left(\ln(y^2 + 1)\right)^{100}$

10. Compute the following derivatives. In addition to rules that you have memorized, they require rules that you have not been asked to memorize. Any necessary rules should be written on the board during the lab.

a) $\frac{d}{dx}\left(\frac{1}{x} - \tan^{-1}(x)\right)$ **b)** $\frac{d}{dt}\left(\tan(t^2) + t^2\cot(t)\right)$ **c)** $\frac{d}{dx}\sin^{-1}(2^x)$

11. Let $f(x) = e^{\pi} \cdot \pi^x \cdot x^{\pi}$. Find $f'(x)$.

12. The following limit is equal to $f'(a)$ for some function $f(x)$ and for some number a. Figure out what $f(x)$ and a are. Then find the value of the limit by computing $f'(x)$ *using derivative rules* (not limits).

$$\lim_{h \to 0}\left(\frac{\sqrt{5 - (2+h)^2} - 1}{h}\right)$$

13. Rewrite $\sin\left(\sin^{-1}(x)\right) + \cos\left(\sin^{-1}(x)\right)$ without using any trigonometric functions.

14. A symbol can represent anything you say it represents. When doing derivatives, you have to work with symbols representing constants and symbols representing variables.

a) Compute $\frac{d}{dx}\left(xe^{-y} + \sin(xz)\right)$, where y and z are constants.

b) Compute $\frac{d}{dy}\left(xe^{-y} + \sin(xz)\right)$, where x and z are constants.

c) Compute $\frac{d}{dz}\left(xe^{-y} + \sin(xz)\right)$, where x and y are constants.

d) Compute $\frac{d}{dt}\left(xe^{-y} + \sin(xz)\right)$, where x, y, and z are constants.

15. The chain rule can be used to prove more general derivative rules than those we have stated. For example, $\frac{d}{dx}\sin(ax) = a\cos(ax)$ for any constant a. Such rules are generally not worth memorizing, since the chain rule can always be applied directly. However, you will often see such more general rules in tables of derivatives. In that spirit, find general rules for the following derivatives:

a) $\frac{d}{dx}\ln(ax + b)$, where a and b are constants.

b) $\frac{d}{dx}(f(x))^n$, for any function $f(x)$, where n is an integer.

c) $\frac{d}{dx}f(e^x)$, for any function $f(x)$.