Calculus II, Spring 2005
Lab 10

Math 131-01
March 29

Exercises to hand in: *Please hand in your responses to the following exercises. You should work on these problems in a group and turn in a single solution for the entire group.*

1. As the textbook notes (page 502), you can use the following identity to compute certain integrals: $\sin(mx)\sin(nx) = \frac{1}{2}\big(\cos((m-n)x) - \cos((m+n)x)\big)$. (Note that if $m = n$, this becomes $\sin^2(nx) = \frac{1}{2}\big(1 - \cos(2nx)\big)$.)

a) Use these identities to show that $\frac{2}{\pi}\int_0^\pi \sin(mx)\sin(nx)\,dx$ is 0 when $n \neq m$ and that it is 1 when $n = m$, where n and m are positive integers.

b) Suppose that $f(x)$ is a function that has the form $f(x) = \displaystyle\sum_{n=1}^{K} a_n \sin(nx)$ for some positive integer N and some constants $a_1, a_2, \ldots, a_K$. Suppose that we know that $f(x)$ can be written in this form, but we don't know the constants a_m. Use the properties of integrals and the results from part **a)** to show that a_n can be computed as $a_m = \frac{2}{\pi}\int_0^\pi f(x)\sin(mx)\,dx$.

(The function $\sin(nx)$ represents a "pure tone" or a "pure frequency" of n. $f(x)$ is a combination of these pure tones. This exercise shows how $f(x)$ can be analyzed to determine how much of each pure frequency it contains. This idea is the beginning of *digital signal analysis*, which is used in computer processing of sound waves and other signals. Extended to infinite sums, it leads to *Fourier analysis*, one of the core fields of applied mathematics.)

2. Use a double integration by parts to evaluate the integral $\int \sin(\ln(x))\,dx$.

3. The following integrals can each be done using a combination of substitution followed by other methods of integration. Compute each integral.

a) $\displaystyle\int \sin(z)\cos(z)\sin(\cos(z))\,dz$ **b)** $\displaystyle\int \cos(z)\sin^3(\sin(z))\,dz$

c) $\displaystyle\int x^3\cos^3(x^4)\sin^2(x^4)\,dx$ **d)** $\displaystyle\int \sin(\sqrt{x})\,dx$

4. **a)** Use algebra to show that $\dfrac{1}{x^3 - x} = \dfrac{1/2}{x+1} + \dfrac{1/2}{x-1} - \dfrac{1}{x}$.

b) Use the result from part **a)** to find the integral $\displaystyle\int \frac{1}{x^3 - x}\,dx$.

c) Use the substitution $u = \sin(\theta)$ and the result from part **b)** to compute the integral

$$\int \frac{1}{\sin(\theta)\cos(\theta)}\,d\theta$$

(We did this integral in a very different way in class, which illustrates that there are often several ways to do the same integral.)

If you have any extra time, please work on problems 1–22 on page 550.