Calculus II, Spring 2005 — Math 131-01
Lab 11 — April 5, 2005

Please hand in your responses to the following exercises. You should work on these problems in a group and turn in a single solution for the entire group.

1. Compute the integrals: **(a)** $\displaystyle\int \frac{(\sin^2(x)+1)\cos(x)}{\sin^3(x)-4\sin^2(x)}\,dx$ and **(b)** $\displaystyle\int \frac{e^x(e^x+1)}{2e^{2x}-3e^x+1}\,dx$.

2. Compute each of the following limits, or say that the limit does not exist:

a) $\displaystyle\lim_{x\to 0}\frac{\tan(3x)}{\tan(2x)}$ **b)** $\displaystyle\lim_{x\to 0}\frac{e^x+1}{e^x-1}$ **c)** $\displaystyle\lim_{x\to\infty}\frac{\ln(x)}{\sqrt{x}}$

d) $\displaystyle\lim_{x\to\infty}\frac{x\sqrt{x}}{1+x}$ **e)** $\displaystyle\lim_{x\to 1}\frac{x-\cos(\pi x)}{\ln(x)}$ **f)** $\displaystyle\lim_{x\to 0^-}\frac{2^x-1}{3^x-1}$

3. In Lab 5, we looked at falling objects and air resistance. In the absence of air resistance, an object falling near the surface of the Earth has acceleration $y''=32$ ft/sec^2. If it falls from rest, then its velocity after t seconds is given by $y'=32t$ ft/sec and the distance fallen after t seconds is $y=16t^2$ ft. With air resistance, acceleration becomes $y''=32-ky'$ for some constant k. In Lab 5, you showed that in this case an object that falls from rest satisfies $y=\frac{32}{k}t+\frac{32}{k^2}(e^{-kt}-1)$. Differentiating this, we see that its velocity is $y'=\frac{32}{k}-\frac{32}{k}e^{-kt}$. The equations are entirely different in the presence of air resistance than they are in its absence! But suppose that the air resistance is very small. Shouldn't the two equations give almost the same result in that case? More specifically, fix a value of t and look at the predictions made by the equations with and without air resistance. If the air resistance is very small, then the predictions should be almost the same.

a) Explain why this argument can be summarized by saying that for any given value of t, $\lim\limits_{k\to 0^+}\left(\frac{32}{k}-\frac{32}{k}e^{-kt}\right)=32t$ and $\lim\limits_{k\to 0^+}\left(\frac{32}{k}t+\frac{32}{k^2}(e^{-kt}-1)\right)=16t^2$.

b) Use L'Hôpital's Rule to show that $\lim\limits_{k\to 0^+}\left(\frac{32}{k}-\frac{32}{k}e^{-kt}\right)=32t$. Before applying L'Hôpital's Rule, check that it applies—you will need to do a little algebra before it does. Note that in this problem t is a constant and k is the variable, so you will be differentiating with respect to k.

c) Use L'Hôpital's Rule to show that $\lim\limits_{k\to 0^+}\left(\frac{32}{k}t+\frac{32}{k^2}(e^{-kt}-1)\right)=16t^2$. Again, you will need to do a bit of algebra to make it apply.

4. L'Hôpital's rule applies to the "indeterminate forms" $0/0$ and ∞/∞. With a little algebra, it can often be used to handle the "indeterminate forms" $0\cdot\infty$, 1^∞, ∞^0, 0^0, and $\infty-\infty$. However, it is not needed for "determinate forms" such as $1/\infty$. This form is determinate in the sense that if $\lim\limits_{x\to a}f(x)=1$ and $\lim\limits_{x\to a}g(x)=\infty$, then we can say that $\lim\limits_{x\to a}\frac{f(x)}{g(x)}=0$ without any further work. This is true because 1 divided by a very large number will be approximately zero. Find five other *determinate* forms and explain in words why each one is determinate. You answer should include a variety of forms—don't just say $2/\infty$, $3/\infty$, $4/\infty$, $5/\infty$, and $6/\infty$!