Calculus II, Spring 2005 — Math 131-01
Lab 12 — April 12, 2005

The third and final in-class test will be given on Friday, April 15. It will cover Chapter 7, Sections 1, 2, 3, 5, 7, and 8. This means that you should be able to compute integrals that use the methods of substitution, integration by parts, and partial fractions, and you should know how to approach integrals involving trigonometric functions, as in Section 7.3. In some cases, you will be told which method of integration to use, but in others you will have to decide on your own which method is appropriate.

You should know when to apply L'Hôpital's Rule, and you should be able to use it for various indeterminate forms including $0/0$, $\infty/infty$, $\infty - \infty$, 1^∞, 0^0, and ∞^0. You should be able to recognize when an integral is improper and you should be able to compute all types of improper integrals.

You should know how to write down the partial fraction decomposition of any rational function, even though time restraints on the test mean that you probably will only be asked to do the integral only in the case where the denominator is a product of linear factors.

I do not expect you to memorize trigonometric identities, except for the most basic ones: $\sin^2(\theta) + \cos^2(\theta) = 1$ and $\tan^2(\theta) + 1 = \sec^2(\theta)$. (Note that the latter can be derived from the former.) You should also know the definitions of all the trigonometric functions and you should know, or be able to figure out, their values at 0, $\pi/2$, and π. You should know the basic derivative formulas (numbers 1–16 on the inside front cover of the textbook) and the most basic integral formulas (numbers 1–8 and 13–16 from the textbook). If any additional formulas or identities are needed for the test, they will be provided to you.

And of course, you should expect one or two essay questions. For example: You should know what it meant by a "form" in L'Hôpital's Rule and what it means for a form to be indeterminate. You should be able to explain how integration by parts follows from the chain rule. You should understand why some integrals are "improper."

This week's lab is practice for the test. There is nothing to hand in. A solution sheet is available.

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1. Use integration by parts to compute the integral $\displaystyle\int \arcsin(x)\,dx$

2. Compute the trigonometric integral $\displaystyle\int \frac{\cos(\theta)^3}{\sin(\theta)}\,d\theta$

3. Compute the integral $\displaystyle\int \sin(x)\sin(2x)\,dx$

4. Use the method of partial fractions to compute $\displaystyle\int \frac{x^2+1}{x(x-1)(x+3)}$

5. Give the partial fractions decomposition of each of the following rational functions:

a) $\dfrac{7-3x}{(x+1)^3(3x-5)}$ **b)** $\dfrac{3x^3+7x-1}{(2x^2+1)(3x^2+1)^2}$ **c)** $\dfrac{x^3}{x^2-1}$

6. Compute the limits

a) $\lim\limits_{x\to\infty} xe^{-x}$ **b)** $\lim\limits_{x\to\infty} xe^{x}$ **c)** $\lim\limits_{x\to\infty} \sin(x)e^{-x}$ **d)** $\lim\limits_{x\to\pi/2} \dfrac{\cos(3x)-1}{\cos(5x)-1}$

7. Compute the limits

a) $\lim\limits_{z\to 1^+} \left(\dfrac{1}{z-1} - \dfrac{1}{z^2-1}\right)$ **b)** $\lim\limits_{x\to 0} \dfrac{\sin(x)-x}{x^3}$ **c)** $\lim\limits_{x\to\infty} (1+x)^{1/x}$

8. Compute the indefinite integrals:

a) $\displaystyle\int x\cos(3x^2)\,dx$ **b)** $\displaystyle\int x\cos(3x)\,dx$

c) $\displaystyle\int \cos(3x)\,dx$ **d)** $\displaystyle\int x^3\cos(3x^2)\,dx$

9. Compute $\displaystyle\int_0^{\pi/2} \sin^2(x)\cos^3(x)\,dx$

10. Compute $\displaystyle\int_0^{\infty} x^3 e^{-x^2}\,dx$

11. Compute $\displaystyle\int_0^{1} \ln(x)\,dx$

12. Compute the improper integrals

a) $\displaystyle\int_e^{\infty} \frac{1}{x(\ln(x))^2}\,dx$ **b)** $\displaystyle\int_e^{\infty} \frac{1}{x\ln(x)}\,dx$

c) $\displaystyle\int_e^{\infty} \frac{1}{x(\ln(x))^{1/2}}\,dx$ **d)** $\displaystyle\int_e^{\infty} \frac{\ln(x)}{x^2}\,dx$