Calculus II, Spring 2005 — Math 131-01

Lab 14 — April 26, 2005

Please hand in your responses to the following exercises. You should work on these problems in a group and turn in a single solution for the entire group. This will be our last graded lab! The final lab next week will be a review for the test.

1. Determine whether each of the following infinite series converges absolutely, converges conditionally, or diverges. Don't forget to fully justify your answers. (One of these series requires about a half-page of justification!)

a) $\displaystyle\sum_{n=1}^{\infty} \frac{5}{\sqrt{n}}$ **b)** $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n^3+1}$ **c)** $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{5^n\sqrt{n}}$ **d)** $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}\sqrt{n}}{n+1}$

e) $\displaystyle\sum_{n=1}^{\infty} \frac{2}{3^n+n^3}$ **f)** $\displaystyle\sum_{n=1}^{\infty} \frac{1}{10000\cdot n}$ **g)** $\displaystyle\sum_{n=1}^{\infty} \frac{\sin(n)}{n^2}$ **h)** $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n 2^n}{n!}$

2. We have seen that the infinite series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$ diverges to ∞, but that it does so very *slowly*. Is it possible that in some sense this series diverges "as slowly as possible?" Consider the infinite series $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n\ln(n)}$. Show that this series also diverges to ∞ by comparing it to the improper integral $\displaystyle\int_2^{\infty} \frac{1}{x\ln(x)}\,dx$. About how large does N have to be, in order to have $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n\ln(n)} \geq 100$? How do the terms in the two series compare, as $n \to \infty$? What does all this say about the rates at which the two series grow towards ∞? Can you find a series that diverges even more slowly than $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n\ln(n)}$? Do you think that there is a series that diverges to ∞ "as slowly as possible?" (Note: This problem requires a substantial answer, including some mathematical work and a lot of words!)