Calculus II, Spring 2005 — Math 131-01
Lab 15 — May 3, 2005

The final exam for this course will take place on Sunday, May 8, at 7:00 PM. The exam is cumulative, with some emphasis on the material that we have covered since the third test. You can expect roughly 40% of the test to be on sequences and series, and material from each of the first three tests will make up roughly 20% of the test. The test will be six or seven pages long and should take most people less than two hours to complete. You should expect at least few questions on the exam that would have been too long and complex to give during a one-hour test. As on the third test, you will be given a copy of the formulas from the inside cover of the text book.

You can look back at previous review labs (Lab 4 and Lab 12) and the information sheet that was handed out for the second test for a reminder about what was covered on the three tests. (See <*http://math.hws.edu/eck/math131*> if you need new copies of any of the labs or of the information sheet.) Here are a few things that will not be on the exam: Simpson's rule, trapezoid rule, arc length, and area of a surface of revolution.

From the material on sequences and series, you should know the six tests for divergence that we have covered: geometric series, p-series, n-th term test, direct comparison test, ratio test, and alternating series test. You should know the difference between absolute and conditional convergence. You should know how to test a series for convergence by comparing it to an improper integral. You should know the sum of a geometric series. The only other series sum that I would expect you to know from memory is the power series for e^x. You should understand power series. You should be able to find the radius of convergence and the interval of convergence of a power series. You should be able to integrate and differentiate a power series. You should know the formula for a Taylor series, and you should be able to find the Taylor series for a given function.

The rest of this lab contains review questions from all parts of the course. These exercises are not to be turned in. An answer sheet will be available during the lab.

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1. Determine whether the following series converge:

a) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n^3+2}$ **b)** $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n^2+2}$ **c)** $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n+2}$

2. Suppose that the function $f(x)$ is defined by $f(x) = \displaystyle\sum_{n=0}^{\infty} \frac{n(x-3)^n}{n^2+1}$. (This series converges for $|x-3| \le 1$.) Find $f^{(5)}(3)$.

3. Let k be any non-negative integer. Use the ratio test to show that the radius of the convergence of the series $\displaystyle\sum_{n=1}^{\infty} \frac{(x-1)^n}{2^n n^k}$ is 2.

4. Find a power series whose interval of convergence is the interval:

a) $(-1,3)$ **b)** $[-1,3]$ **c)** $[-1,3)$ **d)** $(-1,3]$

5. Find the radius of convergence of the power series $\sum_{n=1}^{\infty} \frac{n!x^n}{n^n}$. (Hard problem!)

6. Using geometric series, we can write $\frac{1}{1+x^2} = \frac{1}{1-(-x^2)} = \sum_{n=0}^{\infty}(-x^2)^n = \sum_{n=0}^{\infty}(-1)^n x^{2n}$, for $|x| < 1$. Using this fact, find a power series that converges to $\arctan(x)$.

7. Find the first four terms of the Taylor series for $\sqrt{x}$ centered at 2.

8. State the First Fundamental Theorem of Calculus.

9. Write down a Riemann sum that approximates the integral $\int_1^5 f(x)\,dx$ given the following table of values of $f(x)$ for certain values of x:

x	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0
$f(x)$	1.3	1.6	1.85	2.12	2.4	2.6	2.77	3.0	3.1

10. Compute the following definite and indefinite integrals:

a) $\int \frac{x^2+1}{x^3+3x}\,dx$ **b)** $\int_0^2 2z\sqrt{z^2+1}\,dz$ **c)** $\int x^2 + xe^{2x}\,dx$

d) $\int \sec^2(\theta)\tan(\theta)\,d\theta$ **e)** $\int \frac{5x-3}{2x(x-1)}\,dx$ **f)** $\int_0^1 \ln(x^2+1)\,dx$

11. Compute the derivative $\frac{d}{dx}\int_0^{2x^3} e^{x^2}\,dx$.

12. The region bounded by the curve $y = x^2$ and the line $y = 4$ is rotated about the line $y = -1$. Find the volume of the resulting solid of revolution.

13. The region under the curve $y = \sin(x)$ for $0 \leq x \leq \frac{\pi}{2}$ is rotated about the y-axis. Find the volume of the resulting solid of revolution.

14. The size of a bacterial culture is growing exponentially. After 40 minutes, it has doubled in size. How much longer will it take for it to grow to three times its original size?

15. Solve the differential equation $\frac{dy}{dx} = x\sqrt{4-y^2}$, with the initial condition that $y = 1$ when $x = 0$.

16. Throughout two terms of calculus, you have used *limits* in various ways. In each case, limits bring clarity and rigor to a problem that would be hard to deal with in any other way. Write an essay that describes some of the applications of limits in calculus. In each case, discuss the problem that is solved by using limits, and explain why limits are so essential to the solution.