Calculus II, Spring 2005 Lab 2

Math 131-01
January 25

Exercises to hand in: *Please hand in your responses to the following exercises. You should work on these problems in a group and turn in a single solution for the entire group.*

1. A function $f(x)$ is defined on the interval [1.0,2.6]. The values of $f(x)$ are measured for certain values of x. The results are shown in the following table.

x	1.0	1.2	1.4	1.6	1.8	2.0	2.2	2.4	2.6
$f(x)$	2.7	2.9	2.85	2.75	2.6	2.4	2.1	1.6	0.8

Given the data in this table, what is your **best** estimate for the value of $\int_{1.0}^{2.6} f(x)\,dx$? **Explain your reasoning.**

2. For this problem, you need to be familiar with the graph of the function $y = \sin(x)$. It is a fact that $\int_0^{\pi} \sin(x)\,dx = 2$. Based on this fact and your knowledge of the graph, find the values of each of the following definite integrals. **Explain your reasoning in each case; pictures would help.** (The last one is the hardest.)

a) $\int_0^{2\pi} \sin(x)\,dx$ **b)** $\int_0^{\pi/2} \sin(x)\,dx$ **c)** $\int_0^{3\pi/2} \sin(x)\,dx$

d) $\int_{-1}^{1} \sin(x)\,dx$ **e)** $\int_0^{\pi} 3\sin(x)\,dx$ **f)** $\int_0^{\pi/3} \sin(3x)\,dx$

3. Let $f(x)$ be an **increasing** continuous function on the interval $[a, b]$. Suppose that the interval $[a, b]$ is divided into n equal subintervals. Draw a picture showing both the *lower Riemann sum* and the *upper Riemann sum.* What area(s) in the picture represent the *difference* between the upper and the lower sum? Try to find a simple exact formula for the difference. What happens to the difference as $n \to \infty$? What does this prove about the limit of the upper Riemann sums compared to the limit of the lower Riemann sums as $n \to \infty$? What does it prove about the limit of arbitrary Riemann sums for this function? (You should turn in the picture you drew along with a paragraph giving your response to all these questions. You might not have a complete answer, but you should say what you can. If it helps you to think about the problem, you might want to look at a specific case, such as $f(x) = x^2 + 1$ on the interval $[0, 2]$.)

Suggested Practice Exercises: *Not to be turned in. Here are some odd-numbered problems from the book that would be good practice. Note that answers to odd-numbered problems are in the back of the book, and complete solutions are in the student solutions manual.*

- Section 4.3, # 11, 13, 17, 21, 25, 31, 41, 45