Calculus II, Spring 2005 — Math 131-01
Lab 4 — February 8

There is a **test** in class on Friday. This lab is meant to help you review for the test. All the problems on the lab would be appropriate for the test, although there are probably more "hard" problems here than could ever be on a real test. This lab will not be graded, and there is nothing to turn in. An answer sheet will be available at the end of the lab.

No calculators will be allowed during the test. Note that you will not be required to evaluate numerical expressions. For example, "$(\sin(\pi/3) - 7/3) - (\sin(-\pi/3) - 2/5)$" would be acceptable as a final answer.

The test will cover Chapter 4, Sections 1 to 8. A table of basic integrals is given on the inside front cover of the textbook. You should have memorized numbers 1 though 8 and 13 and 15 in this table. You do not need to memorize the other integrals in this table—if any of them are needed on the test, they will be provided.

You can expect some "essay-type" questions that require a written answer. There might be short essay questions that test your knowledge of definitions or concepts. There might be longer essays that test your understanding of why something is true, why it is important, or how it can be used.

The test will certainly include a number of problems that ask you to evaluate a given definite or indefinite integral. Remember that you have several methods that can be applied to such problems, including the basic formulas, algebraic manipulation, and substitution. You should be able to set up a Riemann sum or a sum for the trapezoid rule. You do not need to memorize Simpson's rule or the error estimate for Simpson's rule or for the trapezoid rule. You should know the limit definition of the definite integral, but I will not ask you to evaluate any complex limits or to find the numerical values of any definite integrals using the limit definition. You should understand the relationship between definite integrals and areas. You should know the major theorems that we have encountered: the two fundamental theorems and the mean value theorem for definite integrals. You should know how to apply these theorems. You should know summation notation ($\sum$), but you do not need to know the formulas for the sum of i, i^2, or i^3. Other things that you should know include: antiderivative, differential equation, initial condition for a differential equation, average value of a function on an interval. There are no proofs on the test.

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1. Find the following indefinite integrals:

a) $\displaystyle\int \cos(6x)\,dx$ **b)** $\displaystyle\int 4t^5(1+\sqrt{t}\,)\,dt$ **c)** $\displaystyle\int \frac{e^u}{e^u+4}\,du$

d) $\displaystyle\int \frac{e^u+4}{e^u}\,du$ **e)** $\displaystyle\int \frac{2\sin(x)-5}{\cos^2(x)}\,dx$ **f)** $\displaystyle\int x\sec^2(x^2)\sqrt[3]{3\tan(x^2)+1}\,dx$

2. Compute the definite integral $\displaystyle\int_1^e \frac{(\ln(x))^2}{x}\,dx$

3. Find the average value of the function $f(x)=\sqrt{x}$ on the interval $[1,5]$.

4. Find an estimate for the definite integral $\int_0^2 \sqrt{x^3+1}\, dx$ using a Riemann sum with $n = 6$ subintervals.

5. Find an estimate for the definite integral $\int_0^2 \sqrt{x^3+1}\, dx$ using the trapezoid rule with $n = 6$ subintervals.

6. Find the derivative: $\dfrac{d}{dx}\left(\int_0^{3x} \sqrt{z^3+1}\, dz\right)$

7. Find the area of the region bounded by the x-axis, the lines $x = 1$ and $x = 3$, and the graph of the function $y = x^2 e^{x^3}$.

8. Find the area of the region bounded by the y-axis and the graph of $y = x^2 - 9$.

9. Apply the integration formula $\displaystyle\int \frac{1}{\sqrt{a^2 - x^2}}\, dx = \arcsin\left(\frac{x}{a}\right) + C$ and the substitution $u = \sqrt{x}$ to find the antiderivative of the function $\dfrac{1}{\sqrt{x}\,\sqrt{4-x}}$.

10. A point that starts with velocity zero and at position zero has an acceleration given by the function $a(t) = e^{-t}$. Find a formula for its position at time t.

11. Use a substitution in the definite integral to verify that $\displaystyle\int_0^1 x f(x^2)\, dx = \frac{1}{2}\int_0^1 f(u)\, du$. Explain your reasoning carefully! Then use this result and a geometric formula to evaluate $\displaystyle\int_0^1 x\sqrt{1-x^4}\, dx$.

12. State the definition of the definite integral $\int_a^b f(x)\, dx$ in terms of a limit.

13. Assume that $b > a$. Why is $\int_a^b |f(x)|\, dx$ always a *non-negative* number?

14. If c is greater than b, is $\int_a^c f(x)\, dx$ necessarily greater than $\int_a^b f(x)\, dx$? Why or why not. (A picture might be helpful to illustrate your answer.)

15. In the proof of the Second Fundamental Theorem of Calculus, we defined $F(x) = \int_a^x f(t)\, dt$, and we used the fact that $\dfrac{F(x+h) - F(x)}{h} = \dfrac{1}{h}\displaystyle\int_x^{x+h} f(t)\, dt$. Explain why this is true.

16. The first part of this course studied two ideas that might seem, at first, to be unrelated: *areas* and *antiderivatives*. The First Fundamental Theorem of Calculus reveals a relationship between these ideas. Discuss this relationship.