

# Calculus II, Spring 2005

## Lab 6

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Math 131-01  
February 22

**Exercises to hand in:** Please hand in your responses to the following exercises. You should work on these problems in a group and turn in a single solution for the entire group. If you have extra time, you can start on this week's homework.

1. There are two wooden models to be passed around in lab today. The base of each solid is a circle. On one solid, cross-sections perpendicular to the base are squares. On the other solid, cross-sections perpendicular to the base are triangles. These are standard calculus examples for finding volumes as integrals of cross-sectional areas. Sometime during the lab, you should get the solids, examine them, and find their volumes (or approximate the volumes as closely as you can). You can use the ruler in the left margin of this page.
2. There is a paper wedding bell to be passed around in lab today. This bell unfolds from a two-dimensional pad into a three-dimensional "solid of revolution". An outline of the pad is traced on the back of this sheet. Use this outline, the ruler in the left margin of this page, and some calculus techniques to estimate the volume of the wedding bell. Explain your reasoning.
3. The equation  $\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1$  defines an ellipse (oval) that is centered at  $(0, 0)$  and stretches from  $-a$  to  $a$  along the  $x$ -axis and from  $-b$  to  $b$  along the  $y$ -axis. Solve the equation for  $y$ . How can the area of the ellipse be represented as the area between two curves? Write the integral that represents the area of the ellipse. Finally, use the substitution  $u = \frac{x}{a}$  and the fact that the area of a circle of radius 1 is  $\pi$  to show that the area of the ellipse is  $\pi ab$ . How does this formula for the area of an ellipse relate the formula for the area of a circle of radius  $r$ ?
4. Let  $r$  and  $h$  be positive constants. Consider the region under the line  $y = \frac{r}{h}x$  on the interval  $[0, h]$ . If this region is rotated around the  $x$ -axis, the result is a cone. The height of the cone is  $h$ , and the radius of its base is  $r$ . Use the "disk method" to show that the volume of such a cone is given by the formula  $\frac{1}{3}\pi r^2 h$ .