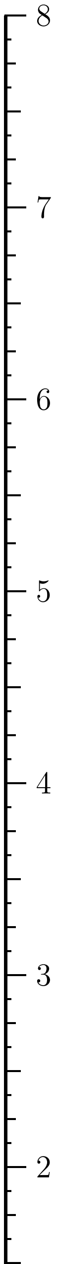
Calculus II, Spring 2005
Lab 7

Math 131-01
March 1



1. Consider the region bounded by the curve $y = \sin(x)$, the curve $y = \sin(x)+3$, the y-axis, and the line $x = 10$. Suppose that this region is rotated about the y=axis. Describe the resulting solid in words. Find the volume of the resulting solid. Why was this problem so easy? What simpler solid has the same volume as the solid in this problem, and why?

2. Consider the triangle with vertices at the points (0,0), (4,0), and (6,3). Suppose that the triangle is rotated about the x-axis.

a) Describe the resulting solid in words.

b) Find the volume of the resulting solid using the "shell" method.

c) Find the volume when the same region is rotated about the y-axis, using whichever method is easier.

3. Suppose that A and B are constants, with $A > B$, Consider the circle with radius B and with center at the point $(A, 0)$. An equation for this circle is $(x-A)^2+y^2 = B^2$. Suppose the region inside this circle is rotated about the y-axis. The result is a doughnut-shaped object that mathematicians like to call a *torus*.

a) Use the "washer" method to find the volume of the torus. The answer will require evaluation of the integral $\int_{-B}^{B} \sqrt{B^2 - y^2}\, dy$. This integral represents the area of a semicircle of radius B, so its value is $\frac{1}{2}\pi B^2$. (This exercise will exercise your algebra skills!)

b) Show that the volume of the torus is equal to the area of the circle multiplied by the distance that the center of the circle travels as it rotates about the y-axis.

4. There is a beautiful result known as the *Theorem of Pappus* which says, essentially, that part **b)** of the previous problem is true not just for circles but for any figure. The only question is what is meant by the "center" of an arbitrary region. The answer is the *centroid*—the point at which the region would balance on the tip of your pencil if you cut it out of a piece of cardboard of uniform thickness. This is all discussed in Section 6.6 of the textbook.

There are some pieces of cardboard and a scissors available. Draw and cut out a cardboard triangle in the shape of the triangle from problem **2**. You can use the ruler on this sheet—to get a good edge, carefully crease the paper along the long vertical line of the ruler. Find the centroid of the triangle by balancing it on the tip of a pencil's eraser. There will be a few suitable pencils in lab, in case you don't have one. (I found the triangle to be too slippery to balance on the tip of the pencil itself.) Measure the x and y coordinates of the centroid.

Use the Theorem of Pappus with the coordinates that you have found for the centroid of the triangle to compute the same two volumes that you found in problem **2**. Since your centroid is only an approximation, the answers won't be exact. How do your answers compare with the exact volumes computed in problem **2**?