Calculus II, Spring 2005 — Lab 8

Math 131-01
March 8

*There is a **test** in class tomorrow, March 9. This lab is a review of material that might be on the test. This lab will not be graded, and there is nothing to turn in. An answer sheet will be available at the end of the lab.*

1. A man finds an old savings bond that was purchased in 1908 for \$25. Since then, it has been earning interest at an annual interest rate r, compounded continuously. Assuming that the bond is now worth \$1542.83, find r.

2. Suppose that out of a 100 gram sample of a certain radioactive isotope, 0.00357 grams have decayed after one year. Find the half-life of the isotope.

3. Find the general solution of the differential equation $2xyy' = 1$. Then find the particular solution which satisfies $y = 3$ when $x = 1$.

4. The function $y = \tan(\frac{\pi}{4}x)$ intersects the line $y = x$ at the points (0,0) and (1,1). Consider the region between $y = \tan(\frac{4x}{\pi})$ and $y = x$ for $0 \le x \le 1$. Set up but **do not evaluate** integrals that give each of the following quantities: **a)** the area of the region; **b)** the volume that is generated when the region is rotated about the x-axis; **c)** the volume that is generated when the region is rotated about the y-axis; **d)** the volume that is generated when the region is rotated about the line $y = 3$; and **e)** the volume that is generated when the region is rotated about the line $x = -1$.

5. Consider the region inside the trapezoid with vertices at (0,0), (1,1), (2,1), and (3,0). Suppose this region is rotated about the y-axis. Use integration to find the volume of the solid that is generated. Use y as the variable of integration. (Would it be possible to find the volume using some geometry and geometric volume formulas?)

6. Suppose that the base of an object is the trapezoid from the previous problem. And suppose that cross sections perpendicular to the y-axis are equilateral triangles. Find the volume of the solid. (The area of an equilateral triangle with base b is $\left(\frac{\sqrt{3}}{4}\right)b^2$.)

7. Suppose that an object moves along a line from $x = 0$ to $x = 10$ and that the force acting on the object in the direction of motion is $3x$ pounds, when the object is at position x. Find the work done on the object.

8. The curve $y = x^3$ for $0 \le x \le 1$ is rotated about the x-axis. Find the area of the surface of revolution that is generated. Set up but do not evaluate an integral that gives the length of the curve.

9. Consider the region R bounded by the curves $y = x^2 - 4$ and $y = 1 - \frac{1}{4}x^2$. Find the number K such that the line $y = K$ divides this area into two sections of equal area.

10. Briefly explain how to derive the "method of shells" formula $V = \int_a^b 2\pi x f(x)\,dx$ for the volume of a solid of revolution.

11. Why does population tend to grow exponentially? Does it always grow exponentially?