

Section 5.1, # 16, 18, 22, 28, 32, 34, 34e

5.1.16 Left and right Riemann Sums for $f(x) = \frac{1}{x}$ on $[1, 5]$, with $m = 4$.

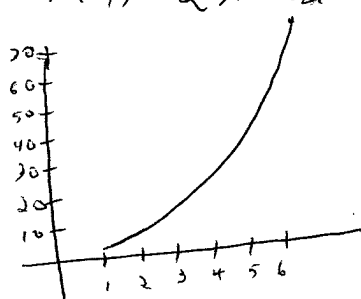
Since $m = 4$, $\Delta x = \frac{b-a}{m} = \frac{5-1}{4} = 1$. The division points are $x_0 = 1$, $x_1 = 2$, $x_2 = 3$, $x_3 = 4$, $x_4 = 5$.

$$\begin{aligned} \text{Left Riemann Sum: } & f(x_0)\Delta x + f(x_1)\Delta x + f(x_2)\Delta x + f(x_3)\Delta x \\ & = f(1) \cdot 1 + f(2) \cdot 1 + f(3) \cdot 1 + f(4) \cdot 1 \\ & = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} = \frac{25}{12} \approx 2.083 \end{aligned}$$

$$\begin{aligned} \text{Right Riemann Sum: } & f(x_1)\Delta x + f(x_2)\Delta x + f(x_3)\Delta x + f(x_4)\Delta x \\ & = f(2) \cdot 1 + f(3) \cdot 1 + f(4) \cdot 1 + f(5) \cdot 1 \\ & = \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} = \frac{77}{60} \approx 1.283 \end{aligned}$$

5.1.18 $f(x) = 2x^2$ on $[1, 6]$, $m = 5$

a)

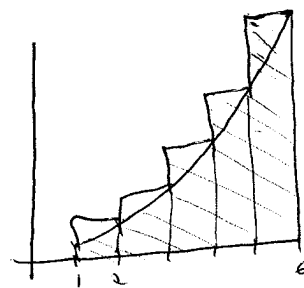
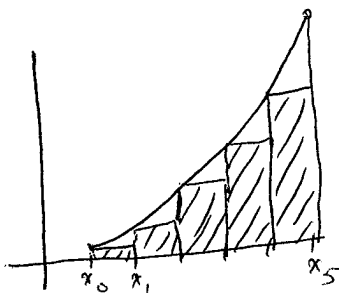


$$b) \Delta x = \frac{6-1}{5} = 1$$

$$x_0 = 1, x_1 = 2, x_2 = 3, x_3 = 4,$$

$$x_4 = 5, x_5 = 6$$

c)



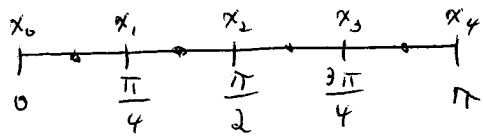
d) Left Riemann Sum

$$\begin{aligned} & f(x_0)\Delta x + f(x_1)\Delta x + \dots + f(x_4)\Delta x \\ & = f(1) \cdot 1 + f(2) \cdot 1 + f(3) \cdot 1 + f(4) \cdot 1 + f(5) \cdot 1 \\ & = 2 \cdot 1^2 + 2 \cdot 2^2 + 2 \cdot 3^2 + 2 \cdot 4^2 + 2 \cdot 5^2 \\ & = 110 \end{aligned}$$

Right Riemann Sum:

$$\begin{aligned} & f(2) \cdot 1 + f(3) \cdot 1 + \dots + f(6) \cdot 1 \\ & = 2 \cdot 2^2 + 2 \cdot 3^2 + 2 \cdot 4^2 \\ & \quad + 2 \cdot 5^2 + 2 \cdot 6^2 \\ & = 180 \end{aligned}$$

5.1.22 Midpoint Riemann Sum for $f(t) = \cos(t/2)$,
on $[0, \pi]$, with $n=4$.



The midpoints are

$$\bar{x}_1 = \frac{\pi}{8}, \bar{x}_2 = \frac{3\pi}{8}, \bar{x}_3 = \frac{5\pi}{8}, \bar{x}_4 = \frac{7\pi}{8}$$

$$\begin{aligned} \text{Riemann Sum: } & f(\bar{x}_1)\Delta x + f(\bar{x}_2)\Delta x + f(\bar{x}_3)\Delta x + f(\bar{x}_4)\Delta x \\ &= f\left(\frac{\pi}{8}\right) \cdot \frac{\pi}{4} + f\left(\frac{3\pi}{8}\right) \cdot \frac{\pi}{4} + f\left(\frac{5\pi}{8}\right) \cdot \frac{\pi}{4} + f\left(\frac{7\pi}{8}\right) \cdot \frac{\pi}{4} \\ &= \cos\left(\frac{\pi}{16}\right) \cdot \frac{\pi}{4} + \cos\left(\frac{3\pi}{16}\right) \cdot \frac{\pi}{4} + \cos\left(\frac{5\pi}{16}\right) \cdot \frac{\pi}{4} + \cos\left(\frac{7\pi}{16}\right) \cdot \frac{\pi}{4} \\ &= \frac{\pi}{4} \cdot \left[\cos\left(\frac{\pi}{16}\right) + \cos\left(\frac{3\pi}{16}\right) + \cos\left(\frac{5\pi}{16}\right) + \cos\left(\frac{7\pi}{16}\right) \right] \\ &\approx \underline{\underline{2.0129}} \end{aligned}$$

5.1.28

x	1	1.5	2	2.5	3	3.5	4	4.5	5
$f(x)$	0	2	3	2	2	1	0	2	3

$\Delta x = \frac{1}{2}$

Left Riemann Sum:

$$\begin{aligned} & f(1) \cdot \frac{1}{2} + f(1.5) \cdot \frac{1}{2} + \dots + f(4.5) \cdot \frac{1}{2} \\ &= 0 \cdot \frac{1}{2} + 2 \cdot \frac{1}{2} + 3 \cdot \frac{1}{2} + 2 \cdot \frac{1}{2} + 2 \cdot \frac{1}{2} + 1 \cdot \frac{1}{2} + 0 \cdot \frac{1}{2} + 2 \cdot \frac{1}{2} \\ &= \frac{1}{2} (0 + 2 + 3 + 2 + 2 + 1 + 0 + 2) \\ &= \frac{1}{2} \cdot 12 = \underline{\underline{6}} \end{aligned}$$

Right Riemann Sum:

$$\begin{aligned} & f(1.5) \cdot \frac{1}{2} + f(2) \cdot \frac{1}{2} + \dots + f(5) \cdot \frac{1}{2} \\ &= 2 \cdot \frac{1}{2} + 3 \cdot \frac{1}{2} + 2 \cdot \frac{1}{2} + 2 \cdot \frac{1}{2} + 1 \cdot \frac{1}{2} + 0 \cdot \frac{1}{2} + 2 \cdot \frac{1}{2} + 3 \cdot \frac{1}{2} \\ &= \frac{1}{2} (2 + 3 + 2 + 2 + 1 + 0 + 2 + 3) \\ &= \frac{1}{2} \cdot 15 = \underline{\underline{\frac{15}{2} = 7.5}} \end{aligned}$$

5.1.32

$$\begin{aligned} \text{a) } 1 + 3 + 5 + 7 + \dots + 99 \\ &= (2 \cdot 0 + 1) + (2 \cdot 1 + 1) + (2 \cdot 2 + 1) + \dots + (2 \cdot 49 + 1) \\ &= \sum_{n=0}^{49} (2n + 1) \end{aligned}$$

$$\begin{aligned} \text{b) } 4 + 9 + 14 + \dots + 44 \\ &= (0 + 4) + (5 + 4) + (10 + 4) + \dots + (40 + 4) \\ &= (0 \cdot 5 + 4) + (1 \cdot 5 + 4) + (2 \cdot 5 + 4) + \dots + (8 \cdot 5 + 4) \\ &= \sum_{i=0}^8 (5i + 4) \end{aligned}$$

$$\begin{aligned} \text{c) } 3 + 8 + 13 + \dots + 63 \\ &= (5 - 2) + (10 - 2) + (15 - 2) + \dots + (60 - 2) \\ &= (5 \cdot 1 - 2) + (5 \cdot 2 - 2) + (5 \cdot 3 - 2) + \dots + (5 \cdot 12 - 2) \\ &= \sum_{k=1}^{12} (5k - 2) \end{aligned}$$

$$\text{d) } \frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \dots + \frac{1}{49 \cdot 50} = \sum_{m=1}^{49} \frac{1}{m(m+1)}$$

5.1.34 d

$$\begin{aligned} \sum_{m=1}^{50} 1 + m^2 &= \left(\sum_{m=1}^{50} 1 \right) + \left(\sum_{m=1}^{50} m^2 \right) \\ &= 50 + \frac{50(50+1)(2 \cdot 50+1)}{6} \\ &= 50 + \frac{50 \cdot 51 \cdot 101}{6} = \underline{\underline{42975}} \end{aligned}$$

5.1.34 e

$$\begin{aligned} \sum_{m=1}^{75} \frac{2m+2}{3} &= \frac{2}{3} \left(\sum_{m=1}^{75} m + 1 \right) \\ &= \frac{2}{3} \left[\left(\sum_{m=1}^{75} m \right) + \left(\sum_{m=1}^{75} 1 \right) \right] \\ &= \frac{2}{3} \left[\frac{75 \cdot (75+1)}{2} + 75 \right] = \underline{\underline{1950}} \end{aligned}$$