MATH 131-01, Spring 2011 **Lab #1, January 20**

Welcome to the first lab of the semester! For the labs in this course, you will work in a group of 3 students (or 4 students ***only*** *if the number of students is not divisible by 3). You might need to meet outside of class to complete the lab. Your group will turn in a single lab report for the group. Solutions must be clearly presented in full English sentences and correct mathematical notation. No credit is given for unsupported answers, and grades will be based partly on presentation. You are required to work together and discuss the problems. Everyone in the group should understand the solutions that you turn in, and everyone in the group is responsible for making sure that everyone else understands. Lab reports will be collected at the beginning of next week's lab, on January 27.*

The first two problems on this lab are ***not to be turned in****. However, I will ask people to present some of them on the board during lab.*

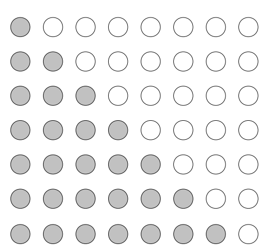
1. Compute the following derivatives

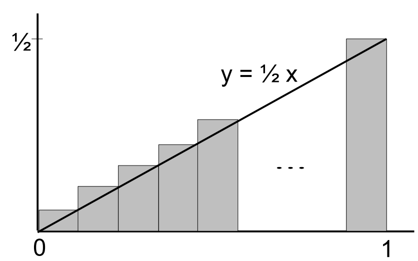
a) $\dfrac{d}{dx}\tan(e^x)$ **b)** $\dfrac{d}{dt}\left(\dfrac{e^{2t}-e^{-2t}}{3t^3+\frac{1}{2}t+7}\right)$ **c)** $\dfrac{d}{dx}\left(x\sin(x^{17})\right)^{17}$

2. Compute the following indefinite integrals. For parts c and d, you have to rewrite the function before you will be able to do the integration.

a) $\displaystyle\int x^{1/2}\,dx$ **b)** $\displaystyle\int \sin(t)-\cos(t)\,dt$ **c)** $\displaystyle\int \frac{1+x}{x\sqrt{x}}\,dx$ **d)** $\displaystyle\int z(1+z)^2\,dz$

3. [To be turned in.] Consider the following illustrations:





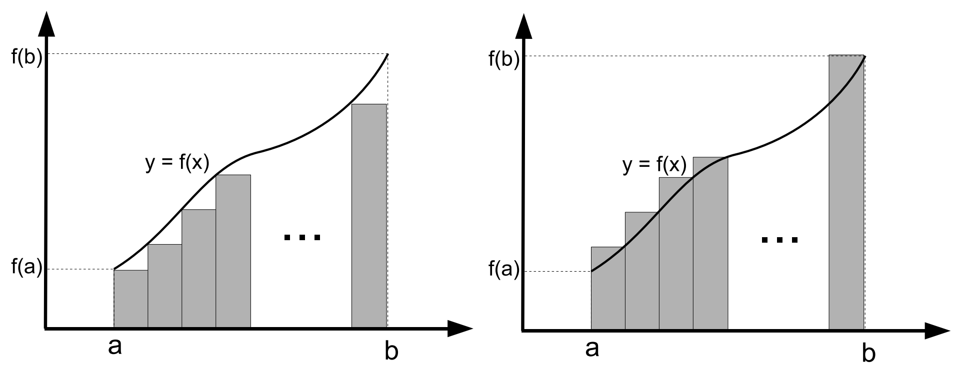
a) Explain why the picture on the left shows that $1+2+3+4+5+6+7=\dfrac{7*(7+1)}{2}$

b) The picture proves a special case of the fact that $1+2+\cdots+n=\dfrac{n(n+1)}{2}$. Explain why the picture makes a convincing case that this fact is true for any positive integer n. You might want to draw pictures for a few more special cases.

c) We have seen that the area under a curve can be computed by dividing the area into n vertical strips and approximating each strip as a rectangle. When you add up the areas of the rectangles, you get a Riemann sum. Based on the picture on the right above, write down a Riemann sum to approximate the area under the function $y = \frac{1}{2}x$ for $0 \leq x \leq 1$, assuming that the number of rectangles is n. (Explain your answer in a paragraph or two. What is the width of each rectangle? Why? What is the height of one of the rectangles? Why? How do you get the sum?)

d) Take the limit of the formula from part **c)**, as $n \to \infty$. You will need the formula from part **b)**. Show your work, explaining your reasoning in words. The region under the curve is a triangle, so you can find its area directly. The limit that you computed should agree with this area. Why should it? Why should the formula from part **b)** be a good approximation for the area when n is large? Answer with a short essay!

4. [To be turned in.] Suppose that $f(x)$ is a function that is defined on the interval $[a, b]$. Assume that f is positive, continuous, and strictly increasing on that interval. Consider a *left* Riemann sum and a *right* Riemann sum for $f(x)$ on $[a, b]$, using the same subintervals for both sums, as shown in the following illustrations:



a) The Riemann sums are approximations for the area under the curve. Explain why the left Riemann sum is definitely an *underestimate* for the area. And explain why the right Riemann sum is definitely an *overestimate* for the area. (This depends on the fact that the function is strictly increasing.)

b) Suppose that the interval $[a, b]$ is broken up into n subintervals, so that the length of each subinterval is $\frac{b-a}{n}$. Let R_n be the value of the right Riemann sum, and let L_n be the value of the left Riemann sum. Find an *exact formula* for the difference $R_n - L_n$. Explain your answer. Draw some pictures to illustrate your explanation. (Hint: Make a picture of the difference between the two areas, and move the pieces around to make a simple geometric figure.)

c) What happens to the difference, $R_n - L_n$, as $n \to \infty$? Use the formula from part **b)** to answer this question.

d) Explain why $\lim_{n\to\infty} R_n$ must equal $\lim_{n\to\infty} L_n$ and why both of these limits must equal the area under the curve.