This week's lab has some exercises on partial fractions and trigonometric substitution. It also has one problem on improper integrals, which we have seen on several previous labs without naming them formally. All of the following problems should be turned in at next week's lab.

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1. Suppose that $\sec(\theta) = \dfrac{x}{2}$. Write the other five trigonometric functions in terms of x.

2. Consider the integral $\displaystyle\int \frac{1}{1-z^2}\,dz$

a) Compute this integral using the method of partial fractions.

b) Compute this integral using the method of trigonometric substitution.

c) Verify that the answers are the same.

3. Use the method of partial fractions to compute $\displaystyle\int \frac{x-2}{x(2x-1)(x+1)^2}\,dx$

4. Compute $\displaystyle\int \frac{1}{\sqrt{x}\sqrt{4+x}}\,dx$ starting with the substitution $z = \sqrt{x}$.

5. Compute $\displaystyle\int \frac{e^x - 1}{e^{2x} - 5e^x + 6}\,dx$ starting with the substitution $z = e^x$.

6. On several labs, we have looked at limits of the form

$$\lim_{b\to\infty} \int_a^b f(x)\,dx$$

In Section 7.7, such limits are officially used as the definition of one kind of *improper integral*, namely

$$\int_a^\infty f(x)\,dx$$

Compute the value of $\displaystyle\int_1^\infty \frac{1}{x^p}\,dx$, where p is a positive constant, and show that the value is infinite for $p \le 1$ and is finite when $p > 1$. Some parts of this have already been done on previous labs, but now you are being asked to do the general case. Note that you will have to consider $p = 1$ as a special case when doing the integral.