

There is a test next Monday, April 11. This lab is practice for the test. There is nothing to hand in. A solution sheet will be available before the end of the lab.

1. Determine whether each of the following sequences converges or diverges. If it converges, find the limit of the sequence as $n \rightarrow \infty$.

a) $\left\{ \frac{3^n}{7^n} \right\}_{n=1}^{\infty}$ b) $\left\{ (-1)^n \frac{n}{n+3} \right\}_{n=0}^{\infty}$ c) $\left\{ \frac{2k+1}{1000000} \right\}_{k=1}^{\infty}$ d) $\left\{ 2 + \frac{(-1)^n}{n} \right\}_{n=1}^{\infty}$

2. You can expect some integration problems on the test that tell you which method to use and others where you have to figure out the method. Find the following integrals, by any method that works:

a) $\int x^3 e^{x^2} dx$ b) $\int x^3 \sin(x^4) dx$ c) $\int \frac{\sin^3(\theta)}{\cos^2(\theta)} d\theta$

3. Compute the improper integral $\int_0^{\infty} x e^{-2x} dx$

4. Suppose that $\{a_k\}_{k=1}^{\infty}$ is a decreasing sequence of *positive* numbers ($a_k > 0$ for all k). This sequence must converge to a number L that satisfies $L \geq 0$. Explain how we know this. Given an example of such a sequence for which the limit is equal to zero, and give an example of such a sequence for which the limit is greater than zero.

5. Use the method of partial fractions to compute $\int \frac{x-1}{(3x-1)(x+2)} dx$

6. Use a trigonometric substitution to compute $\int \frac{8}{(x^2+16)^{3/2}} dx$

7. The radioactive isotope Iodine-131 has a half-life of 8.0197 days. How long will it take for 99% of a sample of Iodine-131 to decay?

8. Define the term *differential equation*, and explain what it means for a function to be a *solution* of a differential equation.

9. Solve the initial value problem $\frac{dy}{dx} = -4x^3 y^2$, $y(0) = \frac{1}{2}$