(1) a) $\sum_{k=1}^{\infty} \frac{5}{\sqrt{k}}$ diverges since $\sum_{k=1}^{\infty} \frac{1}{\sqrt{k}}$ is a p-series with $p<1$.

b) $\sum_{k=1}^{\infty} \frac{(-1)^k \cdot k}{k^3+1}$ converges (absolutely). $\sum_{k=1}^{\infty} \frac{k}{k^3+1}$ converges by the Comparison Test, compared to $\sum_{k=1}^{\infty} \frac{1}{k^2}$: $k^3+1 > k^3$, so $\frac{1}{k^3+1} < \frac{1}{k^3}$ and $\frac{k}{k^3+1} < \frac{k}{k^3} = \frac{1}{k^2}$. Since $\sum_{k=1}^{\infty} \frac{1}{k^2}$ converges, $\sum_{k=1}^{\infty} \frac{k}{k^3+1}$ converges also.

c) $\sum_{k=1}^{\infty} \frac{2^k}{5^k\sqrt{k}}$ converges by the ratio test: $\lim_{k\to\infty} \frac{a_{k+1}}{a_k} = \lim_{k\to\infty} \frac{2^{k+1} \cdot 5^k\sqrt{k}}{5^{k+1}\sqrt{k+1} \cdot 2^k}$

$= \lim_{k\to\infty} \frac{2}{5}\sqrt{\frac{k}{k+1}} = \frac{2}{5}$. Since this limit is <1, the series converges. [Comparison to the convergent geometric series $\sum_{k=1}^{\infty} \left(\frac{2}{5}\right)^k$ also works.]

d) $\sum_{k=2}^{\infty} \frac{\ln k}{k^2}$ converges. For sufficiently large k, $\ln(k) < \sqrt{k}$ (since $\lim_{k\to\infty} \frac{\sqrt{k}}{\ln k} = \lim_{k\to\infty} \frac{\frac{1}{2\sqrt{k}}}{\frac{1}{k}} = \lim_{k\to\infty} \frac{\sqrt{k}}{2} = \infty$). So, $\frac{\ln k}{k^2} < \frac{\sqrt{k}}{k^2}$, That is, $\frac{\ln k}{k^2} < \frac{1}{k^{3/2}}$ for large k. Since $\sum_{k=2}^{\infty} \frac{1}{k^{3/2}}$ is a convergent p-series, $\sum_{k=1}^{\infty} \frac{\ln(k)}{k^2}$ also converges.

e) $\sum_{k=1}^{\infty} \frac{2}{3^k+k^3}$ converges. $3^k + k^3 > 3^k$, and $\frac{2}{3^k+k^3} < \frac{2}{3^k}$. $\sum_{k=1}^{\infty} \frac{2}{3^k}$ is a convergent geometric series. So $\sum_{k=1}^{\infty} \frac{2}{3^k+k^3}$ also converges, by the Comparison Test.

f) $\sum_{k=1}^{\infty} \frac{1}{10000\,k}$ diverges since $\sum_{k=1}^{\infty} \frac{1}{k}$ is a divergent p-series.

g) $\sum_{k=1}^{\infty} \frac{\sin(k)}{k^2}$ converges absolutely. $|\sin k| \le 1$ for all k, so

$\frac{|\sin k|}{k^2} \leq \frac{1}{k^2}$. Since $\sum_{k=1}^{\infty} \frac{1}{k^2}$ is a convergent p-series,

$\sum_{k=1}^{\infty} \frac{|\sin k|}{k^2}$ also converges, by the Comparison Test.

h) $\sum_{k=1}^{\infty} \frac{(-1)^k 2^k}{k!}$ converges absolutely, by the ratio Test. Consider $\sum_{k=1}^{\infty} \frac{2^k}{k!}$.

$$\lim_{k\to\infty} \frac{a_{k+1}}{a_k} = \lim_{k\to\infty} \frac{2^{k+1}}{(k+1)!} \frac{k!}{2^k} = \lim_{k\to\infty} \frac{2}{k+1} = 0.$$

Since this limit is < 1, $\sum_{k=1}^{\infty} \frac{2^k}{k!}$ converges.

2 a) $\sum_{m=0}^{\infty} \frac{x^m}{m!}$. $\lim_{m\to\infty} \left| \frac{a_{m+1}}{a_m} \right| = \lim_{m\to\infty} \left| \frac{x^{m+1}}{(m+1)!} \cdot \frac{m!}{x^m} \right| = \lim_{m\to\infty} \left| \frac{x}{m+1} \right|$

$= |x| \cdot \lim_{m\to\infty} \frac{1}{m+1} = 0$. Since the limit is < 0, the series $\sum_{m=0}^{\infty} \left| \frac{x^m}{m!} \right|$ converges. So $\sum_{m=0}^{\infty} \frac{x^m}{m!}$ converges absolutely, for any x.

b) $\sum_{m=0}^{\infty} \frac{m^2 x^m}{3^m}$. $\lim_{m\to\infty} \left| \frac{a_{m+1}}{a_m} \right| = \lim_{m\to\infty} \left| \frac{(m+1)^2 x^{m+1}}{3^{m+1}} \cdot \frac{3^m}{m^2 x^m} \right|$

$= \lim_{m\to\infty} \left[\left(\frac{(m+1)^2}{m^2} \right) \cdot \frac{1}{3} \cdot |x| \right] = 1 \cdot \frac{1}{3} \cdot |x| = \frac{|x|}{3}$. By the ratio Test, the series converges [absolutely] if $\frac{|x|}{3} < 1$ and diverges if $\frac{|x|}{3} > 1$. For $\frac{|x|}{3} = 1$, i.e. $x = \pm 3$, we have to look at the particular series. For $x = 3$, the series is $\sum_{m=0}^{\infty} \frac{m^2 3^m}{3^m} = \sum_{m=0}^{\infty} m^2$, which diverges. For $x = -3$, it's $\sum_{k=0}^{\infty} \frac{m^2(-3)^m}{3^m} = \sum_{m=0}^{\infty} (-1)^m m^2$, which also diverges. So $\sum_{m=0}^{\infty} \frac{m^2 x^m}{3^m}$ converges if and only if $-3 < x < 3$.

c) $\sum_{m=1}^{\infty} \frac{x^m}{m\, 5^m}$. $\lim_{m\to\infty} \left| \frac{a_{m+1}}{a_m} \right| = \frac{|x|}{5}$, so we get convergence for $\frac{|x|}{5} < 1$, that is $-5 < x < 5$, and divergence for $|x| > 5$.

For $x = 5$, the series is $\sum_{m=1}^{\infty} \frac{5^m}{m\, 5^m} = \sum_{m=1}^{\infty} \frac{1}{m}$, which diverges.

For $x = -5$, the series is $\sum_{m=1}^{\infty} \frac{(-5)^m}{m\, 5^m} = \sum_{m=1}^{\infty} \frac{(-1)^m}{m}$, which converges.

So the power series converges if and only if $-5 \leq x < 5$.