You should turn in all questions from this lab next week at the April 28 lab.
Be sure to justify all your answers carefully.

1. Recall that a series $\sum_{k=1}^{\infty} a_k$ *converges absolutely* if $\sum_{k=1}^{\infty} |a_k|$ converges. It *converges conditionally* if it converges but $\sum_{k=1}^{\infty} |a_k|$ does not converge. The other possibility is that the series $\sum_{k=1}^{\infty} a_k$ diverges—in that case, $\sum_{k=1}^{\infty} |a_k|$ automatically diverges as well.

Determine whether each of the following infinite series converges or diverges; in some cases, it is easiest to show that the series actually converges absolutely:

a) $\displaystyle\sum_{k=1}^{\infty} \frac{5}{\sqrt{k}}$ **b)** $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^n k}{k^3+1}$ **c)** $\displaystyle\sum_{k=1}^{\infty} \frac{2^k}{5^k\sqrt{k}}$ **d)** $\displaystyle\sum_{k=2}^{\infty} \frac{\ln(k)}{k^2}$

e) $\displaystyle\sum_{k=1}^{\infty} \frac{2}{3^k+k^3}$ **f)** $\displaystyle\sum_{k=1}^{\infty} \frac{1}{10000\cdot k}$ **g)** $\displaystyle\sum_{k=1}^{\infty} \frac{\sin(k)}{k^2}$ **h)** $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^k 2^k}{k!}$

(Hint for part **d)**: Show that $\ln(k) < \sqrt{k}$ for all sufficiently large k.)

2. A series such as $\sum_{n=0}^{\infty} \frac{x^n}{n!}$ that contains powers of a variable is called a *power series.*

a) Use the ratio test to show that the power series $\sum_{n=0}^{\infty} \frac{x^n}{n!}$ converges absolutely for all real numbers x. Since the terms of this series are not always positive, you should consider $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right|$.

b) Find all numbers x for which the series $\sum_{n=1}^{\infty} \frac{n^2 x^n}{3^n}$ converges. The ratio test will tell you the convergence of this series for *most* numbers, but there will be two numbers for which the ratio test does not give an answer. For those values of x, you should use some other method to determine whether the series converges.

c) Do the same for $\sum_{n=1}^{\infty} \frac{x^n}{n\cdot 5^n}$. (You will the fact that a certain series, mentioned at the very end of class yesterday, converges conditionally.)