This is the final lab of the semester. It consists of some questions on power series and some review questions on material that has been covered throughout the term. The lab provides some review for the final exam. There is nothing to turn in. An answer sheet will be available near the end of the lab and will be put on-line soon after the end of the lab.

1. The power series $\displaystyle\sum_{k=0}^{\infty} \frac{x^k}{k!}$ converges to e^x for all real numbers x. By substituting x^2 for x, find a power series that converges to e^{x^2} for all real numbers x, and then find a power series that converges to $\displaystyle\int e^{x^2}\,dx$. Finally, write the value of $\displaystyle\int_0^1 e^{x^2}\,dx$ as an infinite series.

2. The power series $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$ converges to $\sin(x)$ for all x. Based on this fact, find the power series that converges to $\cos(x)$.

3. Suppose that the function $f(x)$ is defined by $\displaystyle f(x) = \sum_{n=0}^{\infty} \frac{n(x-3)^n}{n^2+1}$. (This series converges for $|x-3| \le 1$.) Find $f^{(10)}(3)$.

4. Let m be any non-negative integer. Use the ratio test to show that the radius of convergence of the series $\displaystyle\sum_{k=1}^{\infty} \frac{(x-1)^k}{2^k k^m}$ is 2.

5. Find a power series whose interval of convergence is the interval:

a) $(-1, 3)$ **b)** $[-1, 3]$ **c)** $[-1, 3)$ **d)** $(-1, 3]$

6. Suppose that $\{a_k\}_{k=1}^{\infty}$ is a sequence of positive numbers. Is it possible for *both* of the series $\displaystyle\sum_{k=1}^{\infty} a_k$ and $\displaystyle\sum_{k=1}^{\infty} \frac{1}{a_k}$ to converge? Is it possible for *both* of these series to diverge? Justify your answers.

7. Determine whether each of the following series converges or diverges:

a) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{n^4+1}$ **b)** $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{n^3+1}$ **c)** $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{n^2+1}$ **d)** $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{\sqrt{k}}$

8. Compute the derivative $\dfrac{d}{dx}\displaystyle\int_0^{2x^3} e^{t^2}\,dt$.

9. Write down a Riemann sum that approximates the integral $\int_1^5 f(x)\,dx$ given the following table of values of $f(x)$ for certain values of x:

x	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0
$f(x)$	1.3	1.6	1.85	2.12	2.4	2.6	2.77	3.0	3.1

10. Compute the following definite and indefinite integrals:

a) $\displaystyle\int \frac{x^2+1}{x^3+3x}\,dx$ **b)** $\displaystyle\int_0^2 2z\sqrt{z^2+1}\,dz$ **c)** $\displaystyle\int x^2 + xe^{2x}\,dx$

d) $\displaystyle\int \sin^2(x)\cos^3(x)\,d\theta$ **e)** $\displaystyle\int \frac{5x-3}{2x(x-1)}\,dx$ **f)** $\displaystyle\int \ln(x^2+1)\,dx$

11. The region bounded by the curve $y = x^2$, the line $y = 4$, and the y-axis is rotated about the line $y = -1$. Find the volume of the resulting solid of revolution.

12. The region under the curve $y = \sin(x)$ for $0 \le x \le \frac{\pi}{2}$ is rotated about the y-axis. Find the volume of the resulting solid of revolution.

13. The size of a bacterial culture is growing exponentially. After 40 minutes, it has doubled in size. How much longer will it take for it to grow to three times its original size?

14. Solve the differential equation $\frac{dy}{dx} = x\sqrt{1-y^2}$, with the initial condition that $y = 1$ when $x = 0$.

15. What is meant by a *Riemann Sum* of a function $y = f(x)$ on an interval $a \le x \le b$? Discuss how the area under a curve is given by a limit of Riemann sums, and explain whey this should be true. Illustrate your answer with a picture.

16. Throughout two terms of calculus, you have used *limits* in various ways. In each case, limits bring clarity and rigor to a problem that would be hard to deal with in any other way. Write an essay that describes some of the applications of limits in calculus. In each case, discuss the problem that is solved by using limits, and explain why limits are so essential to the solution.