Math 131-01, Spring 2011 Sample Answers to lab #2

(1) a) $\int_0^2 4-x^2\,dx = 4x-\frac{x^3}{3}\Big|_0^2 = \left(4\cdot 2-\frac{2^3}{3}\right)-\left(0\right) = 8-\frac{8}{3} = \underline{\underline{\frac{16}{3}}}$

b) $\int_1^e \frac{1}{x}\,dx = \ln|x|\Big|_1^e = \ln e - \ln 1 = 1-0 = \underline{\underline{1}}$

c) $\int_0^{\pi/2} \sin(\theta)\,d\theta = -\cos\theta\Big|_0^{\pi/2} = -\cos\left(\frac{\pi}{2}\right)-\left(-\cos(0)\right) = 0-(-1) = \underline{\underline{1}}$

d) $\int_{-1}^1 z(1-z)^2\,dz = \int_{-1}^1 z-2z^2+z^3\,dz = \frac{z^2}{2}-\frac{2z^3}{3}+\frac{z^4}{4}\Big|_{-1}^1$

$= \left(\frac{1}{2}-\frac{2}{3}+\frac{1}{4}\right)-\left(\frac{1}{2}+\frac{2}{3}+\frac{1}{4}\right) = \underline{\underline{-\frac{4}{3}}}$

(2) a) $\frac{d}{dx}\int_0^x e^{t^2}\,dt = \underline{\underline{e^{x^2}}}$, direct application of the F.T.o.C.

b) $\frac{d}{dx}\left(\int_x^3 t\sin(t)\,dt\right) = \frac{d}{dx}-\int_3^x t\sin(t)\,dt$, since $\int_b^a = -\int_a^b$

$\neq \cancel{x\sin(x)}$ $= \underline{\underline{-x\sin(x)}}$, by the F.T.o.C.

c) $\frac{d}{dx}\int_1^{x^2}\sqrt{1+t^3}\,dt = \underbrace{\sqrt{1+(x^2)^3}}_{\text{by the F.T.o.C.}} \cdot \underbrace{\frac{d}{dx}(x^2)}_{\text{by the chain rule}} = \underline{\underline{\sqrt{1+x^6}\cdot 2x}}$

<u>OR</u>: Let $w = x^2$

$y = f(w) = \int_1^w \sqrt{1+t^3}\,dt$.

By the chain rule, $\frac{dy}{dx} = \frac{dy}{dw}\cdot\frac{dw}{dx}$

By the F.T.o.C., $\frac{dy}{dw} = f'(w) = \sqrt{1+(w)^3}$

and $\frac{dw}{dx} = 2x$.

So, $\frac{d}{dx}\int_1^{x^2}\sqrt{1+t^3}\,dt = \frac{dy}{dx} = \frac{dy}{dw}\frac{dw}{dx} = \sqrt{1+w^3}\cdot 2x$

$= \sqrt{1+(x^2)^3}\cdot 2x$

$= \underline{\underline{2x\sqrt{1+x^6}}}$

d) $\frac{d}{dx}\int_{-x^2}^{x^3} \cos(t^4)\,dt = \left[\frac{d}{dx}\int_0^{x^3}\cos(t^4)\,dt\right] + \left[\frac{d}{dx}\int_{-x^2}^{0}\cos(t^4)\,dt\right]$,

since $\int_a^b = \int_a^c + \int_c^b$, using $c=0$,

$= \left[\frac{d}{dx}\int_0^{x^3}\cos(t^4)\,dt\right] + \left[\frac{d}{dx}\left(-\int_0^{-x^2}\cos(t^4)\,dt\right)\right]$

since $\int_a^b = -\int_b^a$

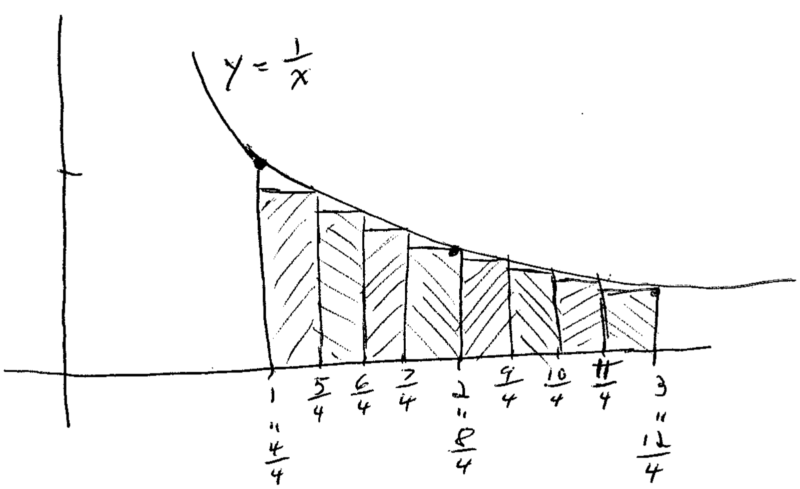
$= \cos((x^3)^4)\cdot\frac{d}{dx}x^3 - \cos((-x^2)^4)\cdot\frac{d}{dx}(-x^2)$

by the F.T.o.C. and the chain rule

$= \cos(x^{12})\cdot 3x^2 - \cos(x^8)\cdot(-2x)$

$= \underline{\underline{3x^2\cos(x^{12}) + 2x\cos(x^8)}}$

(2)



The Right Riemann Sum is the area of the shaded rectangles. This area is completely contained in the area under the curve, so $\int_1^3 \frac{1}{x}\,dx$ is greater than the value of the Riemann Sum.

The Riemann Sum is $f(\frac{5}{4})\cdot\frac{1}{4} + f(\frac{6}{4})\cdot\frac{1}{4} + f(\frac{7}{4})\cdot\frac{1}{4} + \cdots + f(\frac{12}{4})\cdot\frac{1}{4}$

$= \frac{4}{5}\cdot\frac{1}{4} + \frac{4}{6}\cdot\frac{1}{4} + \cdots + \frac{4}{12}\cdot\frac{1}{4}$

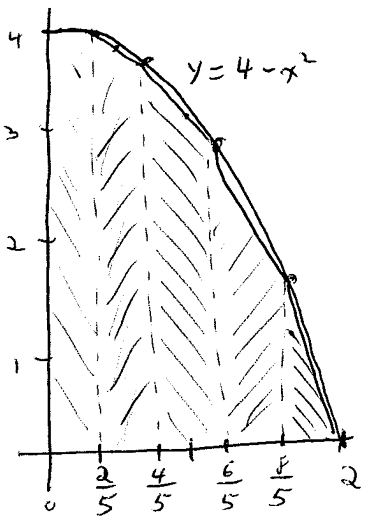
$= \frac{1}{5}+\frac{1}{6}+\frac{1}{7}+\frac{1}{8}+\frac{1}{9}+\frac{1}{10}+\frac{1}{11}+\frac{1}{12}$

$= 1.019877$ [by calculator!]

We see that $\int_1^3 \frac{1}{x}\,dx > 1.019 > 1$. Since $\int_1^B \frac{1}{x}\,dx$ increases as B increases, we see that $\int_1^B \frac{1}{x}\,dx$ must be equal to 1 for some $B < 3$. But e is the number such that $\int_1^e \frac{1}{x}\,dx = 1$,

so $e < 3$.

(4)



$f(x) = 4 - x^2$

$f(0) = 4$

$f(\frac{2}{5}) = 4 - (\frac{2}{5})^2$

$= 4 - \frac{4}{25} = 3.84$

$\vdots$

a) The 5 Trapezoids are shown. [One of them reduces to a triangle.] The division points are $0, \frac{2}{5}, \frac{4}{5}, \frac{6}{5}, \frac{8}{5}, 2$. The sum of the areas of the triangles is

$$\frac{f(0)+f(\frac{2}{5})}{2}\cdot\frac{2}{5} + \frac{f(\frac{2}{5})+f(\frac{4}{5})}{2}\cdot\frac{2}{5} + \frac{f(\frac{4}{5})+f(\frac{6}{5})}{2}\cdot\frac{2}{5}$$

$$+ \frac{f(\frac{6}{5})+f(\frac{8}{5})}{2}\cdot\frac{2}{5} + \frac{f(\frac{8}{5})+f(2)}{2}\cdot\frac{2}{5}$$

$$= \frac{2}{5}\left[\frac{1}{2}f(0) + f(\tfrac{2}{5}) + f(\tfrac{4}{5}) + f(\tfrac{6}{5}) + f(\tfrac{8}{5}) + \frac{1}{2}f(2)\right]$$

$$= \frac{2}{5}\left(\frac{1}{2}\cdot 4 + 3.84 + 3.36 + 2.56 + 1.44 + \frac{1}{2}\cdot 0\right]$$

$$= \cancel{5.264} \quad 5.28$$

This differs from the exact value, 5.33, by 0.05 — which is pretty close considering we used just 5 subintervals.

b) $$\frac{f(x_0)+f(x_1)}{2}\cdot\Delta x + \frac{f(x_1)+f(x_2)}{2}\cdot\Delta x + \frac{f(x_2)+f(x_3)}{2}\Delta x + \cdots + \frac{f(x_{n-1})+f(x_n)}{2}\Delta x$$

$$= \Delta x\left(\frac{1}{2}f(x_0) + f(x_1) + f(x_2) + \cdots + f(x_{n-1}) + \frac{1}{2}f(x_n)\right)$$

[Note that $\frac{f(x_i)}{2}$ occurs twice in the sum, except for $i = 0, i = n$]

c) The average of the left Riemann Sum and right Riemann Sum is

$$\frac{1}{2}\left(\left[f(x_0)\Delta x + f(x_1)\Delta x + \cdots f(x_{n-1})\Delta x\right] + \left[f(x_1)\Delta x + \cdots + f(x_{n-1})\Delta x + f(x_n)\Delta x\right]\right.$$

[Again, $f(x_i)$ occurs Twice, except for $i=0$ and $i=n$.]

$$= \Delta x\left(\frac{1}{2}f(x_0) + f(x_1) + f(x_2) + \cdots + f(x_{n-1}) + \frac{1}{2}f(x_n)\right].$$ Same as Trapezoid rule!