Welcome to the second lab. Problem 1 is for warm-up and is not to be turned in. I will ask people to do some of these on the board after the first 20 minutes or so of the lab.

The remaining problems will be due next week in lab. Remember that you should work together in a group of three students (or possibly four, in a couple of cases). Your group should turn in one lab report for the group.

You are not required to stay in the same group for the entire semester. Today, to shake things up and encourage you to meet new people, we will form groups by alphabetical order. Ordinarily, however, I will not assign people to groups.

1. As a warm-up, compute the values of the following definite integrals, using the Fundamental Theorem of Calculus, Part 2:

a) $\displaystyle\int_0^2 4 - x^2\,dx$ **b)** $\displaystyle\int_1^e \frac{1}{x}\,dx$

c) $\displaystyle\int_0^{\pi/2} \sin(\theta)\,d\theta$ **d)** $\displaystyle\int_{-1}^1 z(1-z)^2\,dz$

2. [To be turned in.] Use the Fundamental Theorem of Calculus, Part 1; properties of definite integrals (Table 329, page 329); and properties of derivatives to compute the following derivatives. State what justifies each step! (Hint: At some points, you will need the chain rule! For example, in part **c)**, you are computing the derivative of a function of the form $F(x^2)$.)

a) $\displaystyle\frac{d}{dx}\int_0^x e^{t^2}\,dt$ **b)** $\displaystyle\frac{d}{dx}\int_x^3 t\sin(t)\,dt$

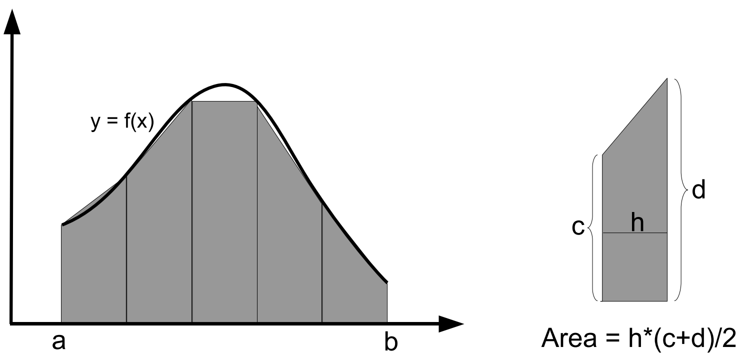
c) $\displaystyle\frac{d}{dx}\int_1^{x^2} \sqrt{1+t^3}\,dt$ **d)** $\displaystyle\frac{d}{dx}\int_{-x^2}^{x^3} \cos(t^4)\,dt$

3. [To be turned in.] From problem **1b)**, we know that $\int_0^e \frac{1}{x}\,dx = 1$. But how do we know the value of e? Use a right Riemann sum with 8 subintervals to show that

$$\int_0^3 \frac{1}{x}\,dx \;>\; 1$$

Explain your answer and explain why this shows that $e < 3$. To illustrate your answer, draw a picture of the area represented by this integral. (Similar computations using larger number of subintervals can give better upper bounds for e. And similar computations using left Riemann sums can give lower bounds. Still, this is not actually the way that the value of e is estimated in practice. For that, one can use an infinite series, as we will see at the end of the semester.)

4. [To be turned in.] In a Riemann sum, the area under the graph of $y = f(x)$ is approximated by rectangles. A better estimate, using the same number of subintervals, can often be obtained with the *trapezoid rule*, as illustrated below on the left. On the subinterval $[x_{i-1}, x_i]$, the area is approximated by a trapezoid with parallel sides of heights $f(x_{i-1})$ and $f(x_i)$. The upper boundary of the shaded region is generated by drawing lines from one point on the curve to the next.



a) Apply the trapezoid rule to estimate the area under the curve $f(x) = 4 - x^2$ on the interval $[0, 2]$, using five subintervals. What is the error in this estimate? We know from problem **1a)** that the exact area is $\frac{16}{3}$, or about 5.333. (You can use a calculator.)

b) The formula for a left Riemann sum is $f(x_0)\Delta x + f(x_1)\Delta x + \cdots + f(x_{n-1})\Delta x$ and for the right Riemann sum is $f(x_1)\Delta x + f(x_2)\Delta x + \cdots + f(x_n)\Delta x$ Find a similar formula for the trapezoid rule for a function $f(x)$ on an interval $[a, b]$, using n subintervals. (Don't try to use summation notation.)

c) Show that the value computed using the trapezoid rule is actually just the average of the left Riemann sum and the right Riemann sum.