*This lab is due next week at the beginning of lab.* ***All*** *problems are to be turned in. Reminder: The first test will be at the end of next week, on Friday, February 11. It will cover Chatper 5.*

**1.** We have defined the definite integral of a function on a closed interval $[a, b]$. The numbers $a$ and $b$ have to be finite. But of course, as usual, we can use limits to investigate what happens as some quantity approaches infinity. Here, for example, we can consider $\lim_{b\to\infty} \int_1^b f(x)\,dx$. Apply this idea in each of the following cases:

**a)** $\lim_{b\to\infty} \left( \int_1^b \frac{1}{x^2}\,dx \right)$ **b)** $\lim_{b\to\infty} \left( \int_1^b \frac{1}{\sqrt{x}}\,dx \right)$ **c)** $\lim_{b\to\infty} \left( \int_1^b \frac{1}{x}\,dx \right)$

**d)** Explain what these limits say in terms of **areas**. What area is represented by the limit, and is that area infinite?

**2.** Suppose that $f$ is a function that is continuous on $[0, \infty]$ and that $\lim_{x\to\infty} f(x) = 1$. Consider the average value of $f(x)$ on the interval $[0, b]$. What do you expect to happen to this average value as $b \to \infty$? That is, what is $\lim_{b\to\infty} \left( \frac{1}{b} \int_0^b f(x)\,dx \right)$? Why? Explain your reasoning. Draw a picture.

**3.** This problem is an exercise in computing indefinite integrals by substitution. We know that $\int x^{11}\,dx = \frac{1}{12}x^{12} + C$. Compute $\int x^{11}\,dx$ using each of the following substitutions, and check that the answer is correct in each case.

**a)** $u = x^6$ **b)** $u = x^3$ **c)** $u = x^2$

**4.** The integral $\int \sin(x)\cos(x)\,dx$ can be done by substitution in two different ways: Let $u = \sin(x)$, or let $u = \cos(x)$. Do the integral **both** ways. Then **explain** why the two answers, which look different, can both be correct.

**5.** Sometimes, an indefinite integral can be found using a substitution that is less than obvious. Compute the following integrals by using the suggested substitution. Then, in each case, **check your answer** by computing its derivative.

**a)** $\int \frac{x^3}{\sqrt{2x^2+1}}\,dx$, using $u = 2x^2 + 1$. (Hint: Write $x^2$ in terms of $u$.)

**b)** $\int \frac{1}{1-\sqrt{x}}\,dx$, using $u = 1 + \sqrt{x}$. (Hint: Solve for $dx$ in terms of $u$ and $du$.)

**c)** $\int \frac{1}{\sqrt{x}\,(1+x)}\,dx$, using $u = \sqrt{x}$. (Hint: Write $x$ in terms of $u$.)

**d)** $\int \sin^3(x)\,dx$, using $u = \cos(x)$. (Hint: Write $\sin^2(x) = 1 - \cos^2(x)$.)