There is a **test** in class tomorrow, February 11. This lab is meant to help you review for the test. This is not a practice test—there are more problems here than fit into a one-hour test. This lab will not be graded, and there is nothing to turn in. An answer sheet is posted on the course web site. A few copies of the solutions will be available at the lab.

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1. The values of a function $f(x)$ are shown in the following table for certain values of x:

x	1.0	1.5	2.0	2.5	3.0	3.5	4.0
$f(x)$	1.5	1.0	1.2	1.7	2.9	3.3	3.6

a) Using this data, write a Riemann sum for $f(x)$ on the interval $[1,4]$, using six subintervals. (There is more than one possible answer.)

b) Sketch the area represented by your Riemann sum.

c) Assume that $f(x)$ is a continuous, non-negative function on $[1,4]$. The definite integral $\int_1^4 f(x)dx$ is defined as a limit of Riemann Sums as the number of subintervals approaches infinity. Explain briefly *why* this limit represents the area under the function $y = f(x)$ on the interval $[1,4]$.

2. Compute the following indefinite integrals:

a) $\displaystyle\int \cos(6x)\,dx$ **b)** $\displaystyle\int 4t^5(1+\sqrt{t})\,dt$ **c)** $\displaystyle\int e^x(4+e^x)\,dx$

d) $\displaystyle\int \frac{(\ln(x))^3}{x}\,dx$ **e)** $\displaystyle\int \frac{\sec^2(x)}{1+\tan(x)}\,dx$ **f)** $\displaystyle\int (x+1)\sqrt{x^2+2x+3}\,dx$

3. Compute the following definite integrals. (For **c**, use geometry!)

a) $\displaystyle\int_1^4 4x^3 - 6x\,dx$ **b)** $\displaystyle\int_0^2 x\sqrt{4-x^2}\,dx$ **c)** $\displaystyle\int_{-2}^2 \sqrt{4-x^2}\,dx$

4. Suppose that $f(x)$ is an integrable function on the interval $[0,5]$, and that

$$\int_0^5 f(x)\,dx = 7 \qquad \text{and} \qquad \int_0^3 f(x)\,dx = 10$$

a) Find the value of $\displaystyle\int_3^5 f(x)\,dx$

b) Is it possible that $f(x) > 0$ for all x? Why or why not?

5. Find the average value of the function $f(x) = \sqrt{x}$ on the interval $[1,5]$.

6. Find the derivative: $\dfrac{d}{dx}\left(\displaystyle\int_0^{3x} \sqrt{z^3+1}\,dz\right)$

7. Assume that $b > a$. Why is $\displaystyle\int_a^b |f(x)|\,dx$ always a *non-negative* number?

8. Write the following summation as an ordinary sum (without using the $\sum$ sign):

$$\sum_{i=1}^{5} \frac{3i}{i^2+1}$$

9. Find an estimate for the definite integral $\displaystyle\int_0^2 \sqrt{x^3+1}\,dx$ using a right Riemann sum with $n = 6$ subintervals.

10. Consider the function $f(x) = 2x - 2$ on the interval $[0, 3]$.

a) Sketch the graph of f on $[0, 3]$, and find the definite integral $\displaystyle\int_0^3 2x - 2\,dx$ geometrically (using areas).

b) Write $\displaystyle\int_0^3 2x - 2\,dx$ as a limit of right Riemann sums, and compute the integral by evaluating that limit.

11. Find the area of the region bounded by the x-axis, the lines $x = 1$ and $x = 3$, and the graph of the function $y = x^2 e^{x^3}$.

12. The Fundamental Theorem of Calculus reveals a relationship between two concepts that at first might appear to be unrelated: *areas* and *antiderivatives.* Write an essay that discusses the relationship, as revealed by the Fundamental Theorem.