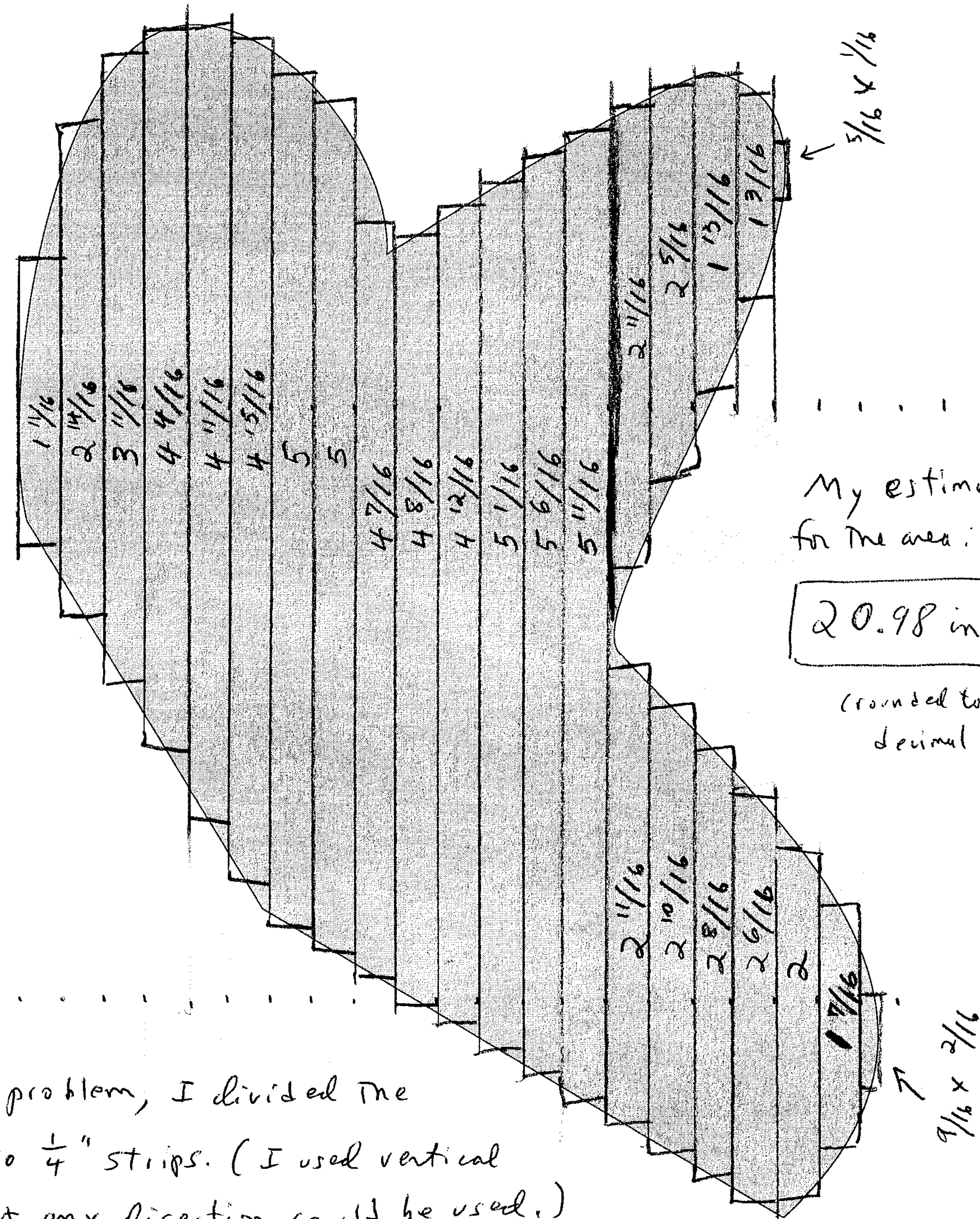
(1)



My estimate for the area:

$\boxed{20.98 \text{ in}^2}$

(rounded to two decimal places)

For this problem, I divided the region into $\frac{1}{4}$" strips. (I used vertical strips, but any direction could be used.)

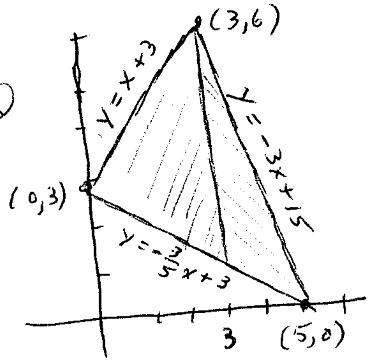
I squared off the strips, using a midpoint rule, and measured the length of each rectangle. (The two small rectangles on the right edge have width less than 1/4". I decided that the best estimate could be obtained by allowing these rectangles to be thinner than the others.)

The estimate for the area is obtained by adding up the areas of all the rectangles. It is equal to

$$\frac{9}{16} \times \frac{2}{16} + \frac{5}{16} \times \frac{1}{16}$$

$$+ \frac{1}{4}\left(\sum \text{lengths of } \tfrac{1}{4}\text{" wide rectangles} \right)$$

(2)



The area must be broken into two regions, by the line $x=3$. The equations of the lines are ~~[illegible]~~ shown.

The area is $\displaystyle\int_0^3 (x+3)-\left(-\tfrac{3}{5}x+3\right)dx + \int_3^5 (-3x+15)-\left(-\tfrac{3}{5}x+3\right)dx$

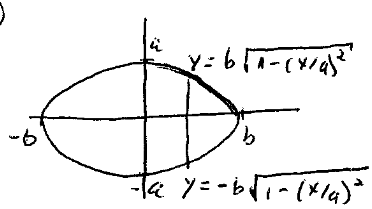
$$= \int_0^3 \frac{8}{5}x\,dx - \int_3^5 -\frac{12}{5}x+12\,dx$$

$$= \left[\frac{8}{5}\cdot\frac{1}{2}x^2\Big|_0^3\right] - \left[-\frac{12}{5}\cdot\frac{1}{2}x^2+12x\Big|_3^5\right]$$

$$= \frac{36}{5} - \left[(-30+60)-\left(-\frac{54}{5}+36\right)\right]$$

$$= \frac{36}{5}+30+\frac{54}{5}-36 = \frac{90}{5}-6 = 18-6 = \underline{\underline{12}}$$

(3)



Solving $\left(\frac{x}{a}\right)^2+\left(\frac{y}{b}\right)^2=1$ gives $y=\pm b\sqrt{1-\left(\frac{x}{a}\right)^2}\,dx$, so

The area is

$w = x/a$, $dw = \frac{1}{a}dx$, $a\,dw = dx$

$x=-b \Rightarrow w = -b/b = -1$

$x = b \Rightarrow w = b/b = 1$

$$A = \int_{-b}^{b} 2b\sqrt{1-\left(\frac{x}{a}\right)^2}\,dx$$

$$= \int_{-1}^{1} 2b\sqrt{1-w^2}\,a\,dw$$

$$= 2ab\int_{-1}^{1}\sqrt{1-w^2}\,dw$$

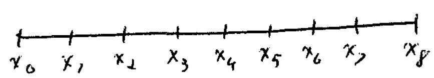
$= 2ab\cdot\frac{1}{2}\pi$, since $\int_{-1}^{1}\sqrt{1-w^2}\,dw$ is the area of a semicircle of radius 1.

$= \pi ab$

In the special case $a=b=R$, this gives πR^2, which is the area of a circle of radius R.

> A circle of radius R can be considered to be an ellipse in which $a=b=R$, and ~~$2ab = 2R\cdot R = 2R^2$~~
> $\pi ab = \pi\cdot R\cdot R = \pi R^2$

(4) a)



On a small piece of wire between x_{i-1} and x_i, the density $\delta(x)$, ~~[illegible]~~ won't change much.

If the density were constant, then $\delta(\bar{x}_i)\Delta x$ would be the exact mass. If the density is almost constant on the piece, then $\delta(\bar{x}_i)\Delta x$ is approximately equal to the mass.

b) $\displaystyle\sum_{i=1}^{n}\delta(\bar{x}_i)\Delta x$ is an approximation for the mass of the whole wire.

This is a Riemann Sum for the function $\delta(x)$ on $[0,\ell]$, where ℓ is the length of the wire. Its limit as $n\to\infty$ is the integral $\int_0^\ell \delta(x)\,dx$.

c) The question is whether the Riemann Sum approximations are good enough so that in the limit, you get the exact value of the mass. We can argue that if ~~[illegible]~~ $\bar{x}_i$ is chosen correctly, then we already have an exact answer in the finite Riemann Sum.