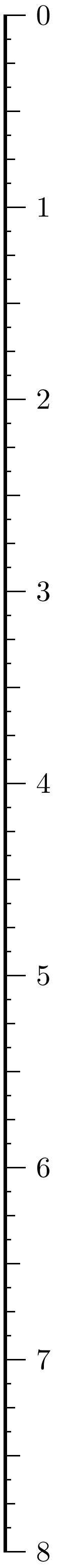
MATH 131-01, Spring 2011 **Lab #5, February 17**

This lab is due next week at the beginning of lab. ***All*** *problems are to be turned in.*



1. Estimate the area, of the region shown on the back of this lab, using the ideas about the area between two curves from Section 6.2 of the textbook. That is, approximate the area using rectangles. You can use the ruler on the right to make the necessary measurements. (One member of the group can make a usable ruler by creasing a lab sheet along the line of the ruler.) Turn in a copy of the figure, with whatever marks and drawing you have made on it. Explain your work in words. You should come up with an actual numerical estimate of the area, in square inches.

2. Consider the triangle with vertices at the points (5,0), (0,3), and (3,6). **Use the methods of Section 6.2** to find the area of this triangle.

3. The equation $\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1$ defines an ellipse (oval) that is centered at $(0,0)$ and stretches from $-a$ to a along the x-axis and from $-b$ to b along the y-axis. Solve the equation for y. How can the area of the ellipse be represented as the area between two curves? Write the integral that represents the area of the ellipse. Finally, use the substitution $u = \frac{x}{a}$ and the formula for the area of a circle to prove that the area of the ellipse is πab. How does this formula for the area of an ellipse relate the formula for the area of a circle of radius r? Your answer should include full explanation and justification, written out in English sentences. Pretend that you are writing a section of a textbook!

4. The *linear density* of a wire can be measured in units of grams per centimeter. This density has the property that if a wire has a length of ℓ centimeters and a linear density of δ grams per centimeter, then the mass of the wire is $\delta\ell$ grams.

Now, suppose that the density of the wire can vary along its length. That is, the density $\delta(x)$ is a function of the distance, x, from one end of the wire (let's say the left end). That is, some parts of the wire have a higher density—more grams per centimeter—than other parts. Can we still compute the mass from the density? We will assume that the density is a continuous function of x.

a) Imagine dividing up the wire into n pieces of equal length. Let Δx be the length of the pieces. Let $\overline{x}_i$ be a point in the i-th piece. Argue that $\delta(\overline{x}_i)\Delta x$ is an approximation for the mass of the i-th piece.

b) Given these approximations for the masses of the pieces, what sum represents an approximation for the mass of the entire wire? What integral do you get when you take the limit of that sum as $n \to \infty$?

c) Can we be sure that the integral from part **b** really does give the mass of the wire? That is, can be be sure that the approximations are good enough that in the limit we get the exactly correct value? Remember that $\overline{x}_i$ can be *any* point in the i-th piece. Argue that there is some $\overline{x}_i$ in the i-th piece such that $\delta(\overline{x}_i)\Delta x$ is actually **equal** to the mass of that piece, and explain why this means that the mass of the wire is given exactly by the integral. (Hint: The density will have some maximum value and some minimum value in the i-th piece.)