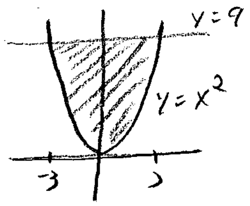
(1) a)



The area of the region is

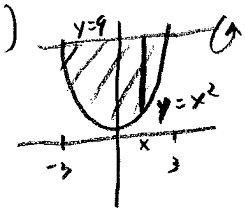
$$A = \int_{-3}^{3} 9 - x^2\,dx = 2\int_0^3 9 - x^2\,dx$$
$$= 2\left(9x - \tfrac{1}{3}x^3 \Big|_0^3\right) = 2\left(27 - \tfrac{1}{3}\cdot 27\right)$$
$$= 2\cdot 18 = \underline{\underline{36}}.$$

To compute the centroid:

$$x_c = \frac{1}{A}\int_{-3}^{3} x\cdot(9-x^2)\,dx = \frac{1}{36}\int_{-3}^{3} 9x - x^3\,dx = 0 \quad \text{(by symmetry)}$$
$$y_c = \frac{1}{A}\int_0^9 y(2\sqrt{y})\,dy = \frac{1}{36}\int_0^9 2y^{3/2}\,dy$$
$$= \frac{1}{36}\cdot 2\cdot\frac{2}{5}y^{5/2}\Big|_0^9 = \frac{1}{45}\cdot 9^{5/2} = \frac{1}{45}\cdot 3^5 = \frac{27}{5} = \underline{\underline{5.4}}$$

So the centroid is $(0, 5.4)$.

b)

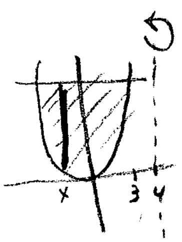


$$\text{Volume} = \int_{-3}^{3} \pi(9-x^2)^2\,dx = 2\pi\int_0^3 (9-x^2)^2\,dx$$
$$= 2\pi\int_0^3 81 - 18x^2 + x^4\,dx = 2\pi\left[81x - 6x^3 + \tfrac{1}{5}x^5\Big|_0^3\right]$$
$$= 2\pi\left[243 - 162 + \tfrac{1}{5}\cdot 243\right] = \frac{1296}{5}\pi = \underline{\underline{259.2\pi}}$$

c) The distance of the centroid from the axis of rotation is $9 - 5.4$, or $\frac{18}{5}$. The centroid moves in a circle of radius $\frac{18}{5}$, so it travels a distance of $2\pi\cdot\frac{18}{5} = \frac{36}{5}\pi$. The area of the region is 36, so by the Theorem of Pappus, the volume is $36\cdot\frac{36}{5}\pi = \frac{1296}{5}\pi = \underline{\underline{259.2\pi}}$

This agrees with the volume computed in part b.

d)



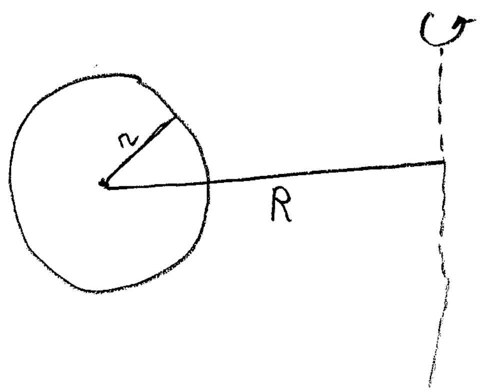
$$\text{Volume} = \int_{-3}^{3} 2\pi(4-x)(9-x^2)\,dx$$
$$= 2\pi\int_{-3}^{3} 36 - 9x - 4x^2 + x^3\,dx$$
$$= 2\pi\left[36x - \tfrac{9}{2}x^2 - \tfrac{4}{3}x^3 + \tfrac{1}{4}x^4\Big|_{-3}^{3}\right]$$
$$= 2\pi\left[\left(108 - \tfrac{81}{2} - 36 + \tfrac{81}{4}\right) - \left(-108 - \tfrac{81}{2} + 36 + \tfrac{81}{4}\right)\right]$$

$$= 2\pi\left[216 - 72\right] = 2\pi(144) = \underline{\underline{288\pi}}$$

e) The distance of the centroid from the axis of rotation is 4, so the centroid travels a distance of $2\pi \cdot 4$, or 8π. The area of the region is 36. By the theorem of Pappus, the volume is $36 \cdot 8\pi = \underline{\underline{288\pi}}$.

This agrees with the answer from part b).

f)



A circle of radius r is rotated about a line that is R units from the center of the circle, producing a "torus". ($R > r$; this is required so the line does not intersect the circle.)

The centroid of the circle is its center. This center travels a distance $2\pi R$ about the axis of rotation. The radius of the circle is πr^2. By the theorem of Pappus, the area of the circle is $\pi r^2 \cdot 2\pi R = \underline{2\pi^2 r^2 R}$