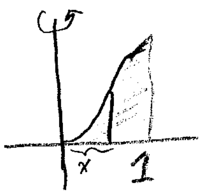
(2) The arc lengths for f and g are given by the integrals $\int_a^b \sqrt{1+(f'(x))^2}\,dx$ and $\int_a^b \sqrt{1+(g'(x))^2}\,dx$. Since $|f'(x)| > |g'(x)|$, we also have $(f'(x))^2 > (g'(x))^2$, $1+(f'(x))^2 > 1+(g'(x))^2$, and $\sqrt{1+(f'(x))^2} > \sqrt{1+(g'(x))^2}$. When we integrate, the integral of the larger function is larger: $\int_a^b \sqrt{1+(f'(x))^2}\,dx > \int_a^b \sqrt{1+(g'(x))^2}\,dx$.

(3) $y=9-x^2$ intersects the x-axis when $9-x^2=0$, that is when $x=\pm 3$. It lies above the x-axis for $-3 \le x \le 3$. The arc length is $\int_{-3}^{3} \sqrt{1+(f'(x))^2}\,dx = \int_{-3}^{3}\sqrt{1+(-2x)^2}\,dx = \int_{-3}^{3}\sqrt{1+4x^2}\,dx$, since $f(x)=9-x^2$ and $f'(x)=-2x$.

(4)



Shells about the y-axis: $\int_0^1 2\pi r h\,dx = \int_0^1 2\pi x \sin(x^2)\,dx$.

$\int 2\pi x \sin(x^2)\,dx = \int \pi \sin(w)\,dw$, where $w=x^2$,

$= -\pi\cos(w) = -\pi\cos(x^2)$

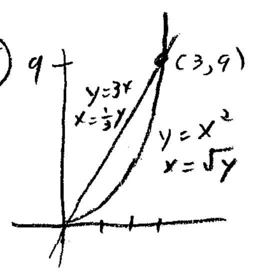
$y=\sin(x^2)$
$x^2=\sin^{-1}(y)$
$x=\sqrt{\sin^{-1}(y)}$

So $\int_0^1 2\pi x\sin(x^2)\,dx = -\pi\cos(x^2)\Big|_0^1 = -\pi(\cos(1)-\cos(0))$

$= -\pi(\cos(1)-1) = \underline{\pi(1-\cos(1))} \approx 1.444$.

(other integrals are $\int_0^1 \pi(\sin(x^2))^2\,dx$, $\int_0^{\sin(1)} 2\pi y\,(1-\sqrt{\sin^{-1}(y)})\,dy$,
and $\int_0^{\sin(y)} (\pi\cdot 1^2 - \pi\cdot(\sqrt{\sin^{-1}(y)})^2)\,dy$.)

(5)



a) $\int_0^3 3x-x^2\,dx$, $\int_0^9 \sqrt{y}-\frac{1}{3}y\,dy$

b) $\int_0^3 \pi(3x)^2-\pi(x^2)^2\,dx$, $\int_0^9 2\pi y\,(\sqrt{y}-\frac{1}{3}y)\,dy$

c) $\int_0^3 2\pi x(3x-x^2)\,dx$, $\int_0^9 \pi(\sqrt{y})^2-\pi(\frac{1}{3}y)^2\,dy$

d) $\int_0^3 2\pi(3-x)(3x-x^2)\,dx$, $\int_0^9 \pi(3-\sqrt{y})^2-\pi(3-\frac{1}{3}y)^2\,dy$

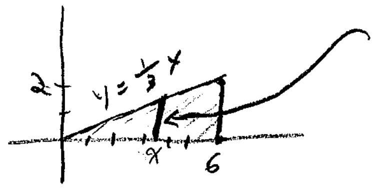
e) $\int_0^3 \pi(3x+1)^2-\pi(x^2+1)^2\,dy$, $\int_0^9 2\pi(y+1)(\sqrt{y}-\frac{1}{3}y)\,dy$

Actually: $3-(-1)$ $x-(-1)$ $y-(-1)$

remember: [horizontal line through region: dy
[vertical line through region: dx

[line parallel to axis of rotation: shells
[line perpendicular to axis of rotation: disks/washers

(6)



This is one side of the square
that is the cross section at x.
The length of this line is $\frac{1}{3}x$,
so the cross-sectional area is $A(x) = \left(\frac{1}{3}x\right)^2 = \frac{1}{9}x^2$.

The volume is $\int_0^6 A(x)\,dx = \int_0^6 \frac{1}{9}x^2\,dx$

$$= \frac{1}{9}\cdot\frac{1}{3}x^3 \Big|_0^6 = \frac{1}{27}\cdot 6^3 = \underline{8}$$

(7) The general formula is $f(b) = f(a) + \int_a^b f'(x)\,dx$,
so here, $f(42) = f(17) + \int_{17}^{42} f'(x)\,dx = \underline{f(17) + \int_{17}^{42} 3e^{-x^2}\,dx}$

(8) $\text{Mass} = \int_a^b \rho(x)\,dx = \int_0^{20} 3 + \sin(x/5)\,dx$

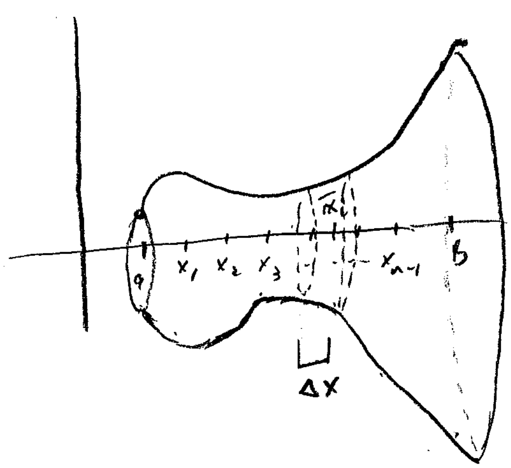
$$= 3x - 5\cos(x/5)\Big|_0^{20}$$

$$= \left[3\cdot 20 - 5\cos(20/5)\right] - \left[0 - 5\underbrace{\cos(0)}_{=1}\right]$$

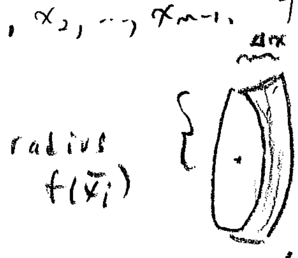
$$= 60 - 5\cos(4) + 5$$

$$= \underline{65 - 5\cos(4)}$$

(9)



Cut the solid into slices at
$x_1, x_2, \ldots, x_{n-1}$. The i-th slice:



is approximately a cylinder
whose radius is $f(\bar{x}_i)$ and
whose height, or thickness, is Δx.
Its volume is about $\pi(f(\bar{x}_i))^2\Delta x$,
the area of the base times the
height. Adding the approximate
volumes of the n slices gives

$$\sum_{i=1}^{n} \pi(f(\bar{x}_i))^2\Delta x$$

(As $n\to\infty$, the approximations
approach the exact value, so
the volume is exactly given by
$\int_a^b \pi(f(x))^2\,dx$.)