There is a **test** coming up on Wednesday of next week. The test will cover Chapter 6, Sections 1 through 6. This lab is review, and most of it will not be turned in. For your lab report next Thursday, you should turn in your answer to Question 1 only. An answer sheet for the remaining questions will be available near the end of the lab today.

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1. [To be turned in.] The *centroid* of a finte region in the plane is the center point of the region in the sense that if you were to cut a copy of the region out of a sheet of material uniform density, then the centroid would be the center of mass of that region—the point at which the region would exactly balance on the tip of your finger. If the region extends from a to b in the x-direction, and from c to d in the y-direction, then the x- and y-coordinats of the centroid are given by the formulas

$$x_c = \frac{\int_a^b x \cdot h(x)\,dx}{A} \qquad\qquad y_c = \frac{\int_c^d y \cdot w(y)\,dy}{A}$$

where A is the area of the region, $h(x)$ is the length of the vertical line through the region at x and $w(y)$ is the length of the horizontal line through the region at y.

The Theorem of Pappus says the when a region is rotated about a line that does not intersect the region, then the volume of the solid that is generated is equal to the area of the region multiplied by the distance traveled by the centroid as it rotates about the line.

a) Consider the region bounded by $y = x^2$ and the line $y = 9$. Find the centroid of this region.

b) Find the volume that is generated when this region is rotated about the line $y = 9$, using the method of disks.

c) Compute the same volume using the Theorem of Pappus, and check that the answer is the same as the value found in part **b)**.

d) Now, suppose the the region is rotated about the line $x = 4$. Use the method of shells to compute the value of the resulting solid.

e) Compute the same volume using the Theorem of Pappus, and check that the answer is the same as the value found in part **b)**.

f) The centroid of a disk is at the center of the disk. Suppose that that a disk of radius r is rotated about a line that is R units from the center of the disk, where $R > r$. The resulting doughnut-shaped solid is called a "torus." Use the Theorem of Pappus to find the volume of the torus, in terms of r and R.

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2. Suppose that f and g are differentiable functions on an interval $[a, b]$, and suppose that $|f'(x)| \geq |g'(x)|$ for all x in $[a, b]$. Explain why the length of the curve $y = f(x)$ on that interval is greater than the length of the curve $y = g(x)$ on the same interval. (Use the integral formula for arc length.)

3. Set up, but do not evaluate, an integral that gives the length of the portion of the curve $y = 9 - x^2$ that lies above the x-axis.

4. Consider the region under the curve $y = \sin(x^2)$ for $0 \leq x \leq 1$. This region can be rotated about either the x-axis or about the y-axis. For each of these directions, the volume of the resulting solid can be found using either the method of shells or the method of disks/washers. *One* of the four resulting integrals can actually be computed (using substitution). Figure out which integral that is, and find the volume.

5. Consider the region in the first quadrant bounded by $y = 3x$ and $y = x^2$. Set up **two** integrals that compute each of the following. One integral should use integration with respect to x, and one should use integration with respect to y. Do **not** evaluate the integrals.

a) The area of the region.

b) The volume of the solid generated when the region is rotated about the x-axis.

c) The volume of the solid generated when the region is rotated about the y-axis.

d) The volume of the solid generated when the region is rotated about the line $x = 3$.

e) The volume of the solid generated when the region is rotated about the line $y = -1$.

6. The base of a solid object is the triangle with vertices at the points (0,0), (6,0), and (6,2). Every cross section perpendicular to the x-axis is a square. Set up, but do not evaluate, an integral that gives the volume of the solid.

7. Suppose that f is a differentiable function with $f(17) = 42$ and $f'(x) = 3e^{-x^2}$ for all x. Write a formula, including a definite integral, for $f(42)$. (Do not try to evaluate it.)

8. Suppose that a wire is 20 centimeters long and the linear density of the wire is given by $\rho(x) = 3 + \sin(x/5)$ grams/centimeter, where x is the distance from one end of the wire, in centimeters. What is the mass of the wire?

9. Suppose that $f(x) > 0$ on the interval $[a, b]$, and consider the solid that is generated when the region under $y = f(x)$, for $a \leq x \leq b$, is rotated about the x-axis. Suppose that the interval is divided into n equal subintervals. Explain *why* the Riemann Sum

$$\sum_{i=1}^{n} 2\pi (f(\overline{x}_i))^2 \, \Delta x$$

is a reasonable approximation for the volume of the solid, and explain how this leads to the integral formula for the "disk method." Illustrate your answer with at least one picture.