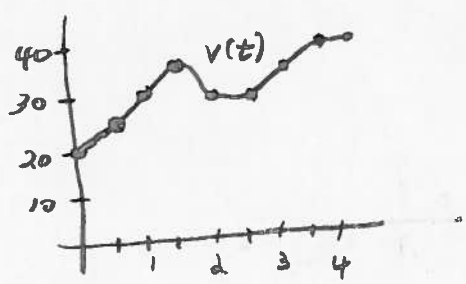
Section 5.1

(18) The left Riemann Sum with $n=4$ is $f(x_1^*)\Delta x + f(x_2^*)\Delta x + f(x_3^*)\Delta x + f(x_4^*)\Delta x$, where $\Delta x = \frac{5-1}{4} = 1$ and x_i is the left endpoint of the ith subinterval. Here, we see that $x_i^* = i$. So the sum becomes $f(1)\cdot 1 + f(2)\cdot 1 + f(3)\cdot 1 + f(4)\cdot 1$. Since $f(x) = \frac{1}{x}$, the sum is $\underline{\underline{1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}}}$.

The right Riemann sum is similar, except that the values of x at the right endpoints are 2, 3, 4, 5, so the sum is $\underline{\underline{\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5}}}$.

(38) 

For $n=2$, $\Delta x = 2$, the intervals are $[0,2]$ and $[2,4]$, and the midpoints are $x_1^* = 1$, $x_2^* = 3$. The Riemann Sum is $v(x_1^*)\Delta x + v(x_2^*)\Delta x = v(1)\cdot 2 + v(3)\cdot 2$
$= \underline{\underline{30\cdot 2 + 35\cdot 2}}$

For $n=4$, $\Delta x = 1$, the midpoints are 0.5, 1.5, 2.5 and 3.5 and the sum is $v(0.5)\cdot 1 + v(1.5)\cdot 1 + v(2.5)\cdot 1 + v(3.5)\cdot 1$
$= \underline{\underline{25 + 35 + 30 + 40}}$

(40) a) $1+3+5+7+\cdots+99 = \sum_{n=1}^{50} (2n-1)$, because
$2\cdot 1 - 1 = 1$, $2\cdot 2 - 1 = 3$, $2\cdot 3 - 1 = 5$, $\ldots$, $2\cdot 50 - 1 = 99$.

b) $4+9+14+\cdots+44 = \sum_{k=0}^{8} 5k+4$, because
$5\cdot 0 + 4 = 4$, $5\cdot 1 + 4 = 9$, $5\cdot 2 + 4 = 14$, $\ldots$, $5\cdot 8 + 4 = 44$

c) $3+8+13+\cdots+63 = \sum_{j=1}^{13} 5j-2$, because
$5\cdot 1 - 2 = 3$, $5\cdot 2 - 2 = 8$, $5\cdot 3 - 2 = 13$, $\ldots$, $5\cdot 13 - 2 = 63$

d) $\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+\cdots+\frac{1}{49\cdot 50} = \sum_{i=1}^{49}\frac{1}{i\cdot(i+1)}$, clearly

(57) $\Delta x = \frac{11-3}{32} = \frac{4}{32} = \frac{1}{8}$. From the definition on page 341, the midpoint Riemann sum is $\sum_{k=1}^{n} f(x_k^*)\Delta x$ where $x_k^* = a + (k-\frac{1}{2})\Delta x$, so in this case, $\sum_{k=1}^{32} f(3+(k-\frac{1}{2})\cdot\frac{1}{4})\cdot\frac{1}{4} = \sum_{k=1}^{32}(3+\frac{1}{4}k-\frac{1}{8})^3\cdot\frac{1}{4}$

$= \frac{1}{4}\cdot\sum_{k=1}^{32}(2.875+\frac{k}{4})^3 = \frac{1}{4}\left[\sum_{k=1}^{32}\left(2.875^3+3\cdot 2.875^2\cdot\frac{k}{4}+3\cdot 2.875\left(\frac{k}{4}\right)^2+\left(\frac{k}{4}\right)^3\right)\right]$

$= \frac{1}{4}\left[\left(\sum_{k=1}^{32}2.875^3\right)+\left(\frac{3\cdot 2.875^2}{4}\sum_{k=1}^{32}k\right)+\left(\frac{3\cdot 2.875}{16}\sum_{k=1}^{32}k^2\right)+\left(\frac{1}{64}\sum_{k=1}^{32}k^3\right)\right]$

$= \frac{1}{4}\left[32\cdot 2.875^3+\frac{3\cdot 2.875^2}{4}\cdot\frac{32(33)}{2}+\frac{3\cdot 2.875}{16}\cdot\frac{32(33)(65)}{6}+\frac{1}{64}\frac{32^2\cdot 33^2}{4}\right]$

The last step uses Theorem 5.1. This would be OK as a final answer, but a calculator shows it's equal to 3639.125, which agrees with the answer in the back of the book.

(62) The formula $\sum_{k=1}^{8} f(1.5+\frac{k}{2})\cdot\frac{1}{2}$ looks like the Riemann sum $\sum_{k=1}^{n} f(x_k^*)\Delta x$ with $n=8$, $\Delta x=\frac{1}{2}$, and $x_k^* = 1.5+\frac{k}{2} = 1.5+k\Delta x$.

There are 8 subintervals of length $\frac{1}{2}$, so the whole interval has length 4. We know $x_1^* = 1.5+\frac{1}{2} = 2$ must be somewhere in the 1st interval. If we say that 2 is the left endpoint of the 1st subinterval, then the whole interval would be $[2,6]$, and the sum would be a left Riemann sum. So: $\sum_{k=1}^{8} f(1.5+\frac{k}{2})\cdot\frac{1}{2}$ is a left Riemann sum for f on the interval $[2,6]$ with $n=\underline{8}$.

(It could also be a right Riemann sum on the interval $[1.5, 5.5]$ or even a midpoint sum on $[1.75, 5.75]$.)