Section 6.1

(8) a) $(0,2)$ and $(4,6)$, because its velocity is negative on those intervals

b) $\int_2^6 v(t)\,dt$, which is $14-10$ or $\underline{\underline{4}}$

c) $\int_0^6 |v(t)|\,dt$, which is $20+14+10$ or $\underline{\underline{44}}$ [Ambiguous, could be $\int_0^6 v(t)\,dt = 6$]

d) $\int_0^8 v(t)\,dt$, which is $-20+14-10+6$ or $\underline{\underline{-10}}$

e) It has moved backwards, in the negative direction, by 10 units.

(30) $v(t) = v(0) + \int_0^t a(x)\,dx = 60 + \int_0^t e^{-x}\,dx$

$$= 60 + \left(-e^{-x}\Big|_0^t\right) = 60 + \left(-e^{-t} + e^0\right)$$

$$= 60 - e^{-t} + 1$$

$$= \underline{\underline{61 - e^{-t}}}$$

$s(t) = s(0) + \int_0^t v(x)\,dx = 40 + \int_0^t 61 - e^{-x}\,dx$

$$= 40 + \left(61x + e^{-x}\Big|_0^t\right) = 40 + 61t + e^{-t} - e^0$$

$$= \underline{\underline{39 + 61t + e^{-t}}}$$

(34) $v(t) = v(0) + \int_0^t a(x)\,dx = 0 + \int_0^t \frac{2x}{(x^2+1)^2}\,dx$

Find $\int \frac{2x}{(x^2+1)^2}\,dx$. Let $w = x^2+1$, $dw = 2x\,dx$

$$= \int w^{-2}\,dw$$

$$= -w^{-1} + C = \frac{-1}{x^2+1} + C$$

So $v(t) = \int_0^t \frac{2x}{(x^2+1)^2}\,dx = \frac{-1}{x^2+1}\Big|_0^t = \frac{-1}{t^2+1} - (-1) = \underline{\underline{1 - \frac{1}{t^2+1}}}$

$a(t) = a(0) + \int_0^t v(x)\,dx = 0 + \int_0^t 1 - \frac{1}{x^2+1}\,dx$

$$= x - \tan^{-1}(x)\Big|_0^t = \left(t - \tan^{-1}(t)\right) - (0 - 0)$$

$$= \underline{\underline{t - \tan^{-1}(t)}}$$

(40) Given $P(0) = 55$ and $P'(t) = 20 - t/5$ for $0 \le t \le 200$.

a) Find $P(6)$.

$$P(6) = P(0) + \int_0^6 P'(x)\,dx$$
$$= 55 + \int_0^6 20 - \tfrac{1}{5}x\,dx$$
$$= 55 + \left(20x - \tfrac{1}{5}\cdot\tfrac{1}{2}x^2 \Big|_0^6 \right)$$
$$= 55 + \left(20\cdot 6 - \tfrac{1}{10}\cdot 6^2 - 0 \right)$$
$$= 55 + 120 - 3.6 = \underline{\underline{171.4}}$$

b)
$$P(t) = P(0) + \int_0^t P'(x)\,dx$$
$$= 55 + \int_0^t 20 - \tfrac{1}{5}x\,dx$$
$$= 55 + \left(20x - \tfrac{1}{10}x^2 \Big|_0^t \right)$$
$$= \underline{\underline{55 + 20t - \tfrac{1}{10}t^2}}$$

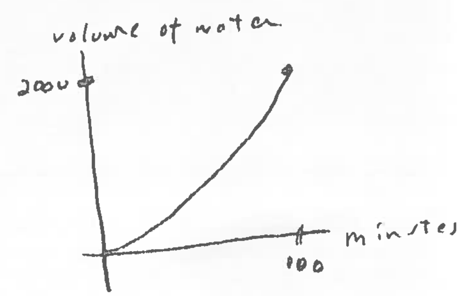
(60) $Q(0) = 0$ and $Q'(t) = 3\sqrt{t}$. [Water flows into a 2000-liter Tank.]

a) Quantity of water in 1st hour (That is after 60 minutes)

$$Q(60) = Q(0) + \int_0^{60} Q'(t)\,dt = 0 + \int_0^{60} 3\sqrt{t}\,dt$$
$$= 3\cdot \frac{t^{3/2}}{3/2}\Big|_0^{60} = 2t^{3/2}\Big|_0^{60} = \underline{\underline{2\cdot 60^{3/2} \text{ liters}}}$$

b) $Q(t) = Q(0) + \int_0^t Q'(x)\,dx = \int_0^t 3x^{1/2}\,dx = 2x^{3/2}\Big|_0^t = \underline{\underline{2t^{3/2}}}$

c) The Tank is full when $Q(t) = 2000$, so solve



$$2t^{3/2} = 2000$$
$$t^{3/2} = 1000$$
$$t = 1000^{2/3} = \left(\sqrt[3]{1000}\right)^2$$
$$= 10^2 = \underline{\underline{100}} \text{ minutes}$$