Math 131-03, Spring 2017 — Sample Answers to Homework # 7

Section 7.1

(28) $$\int \frac{3x+1}{\sqrt{4-x^2}}\,dx = \int \frac{3x}{\sqrt{4-x^2}}\,dx + \int \frac{1}{\sqrt{4-x^2}}\,dx = \underline{\underline{-3\sqrt{4-x^2} + \sin^{-1}\left(\frac{x}{2}\right) + C}}$$

$$\int \frac{3x}{\sqrt{4-x^2}}\,dx = -\frac{3}{2}\int w^{-1/2}\,dw = -\frac{3}{2}\cdot 2w^{1/2} + C = -3\sqrt{4-x^2} + C$$

Let $w = 4-x^2$, $dw = -2x\,dx$

(30) $$\int_2^4 \frac{x^2+2}{x-1}\,dx = \int_2^4 \frac{x^2-x+x-1+3}{x-1}\,dx = \int_2^4 x+1+\frac{3}{x-1}\,dx$$

$$= \frac{x^2}{2}+x+3\ln|x-1|\,\Big|_2^4 = \left(\frac{16}{2}+4+3\ln 3\right) - \left(\frac{4}{2}+2+3\underbrace{\ln 1}_{=0}\right)$$

$$= \underline{\underline{8+3\ln 3}}$$

(33) $$\int \frac{dx}{x^2-2x+10} = \int \frac{1}{x^2-2x+1+9}\,dx = \int \frac{1}{(x-1)^2+9}\,dx$$

Let $w = x-1$, $dw = dx$

$$= \int \frac{1}{w^2+3^2}\,dw = \frac{1}{3}\tan^{-1}\left(\frac{w}{3}\right)+C = \underline{\underline{\frac{1}{3}\tan^{-1}\left(\frac{x-1}{3}\right)+C}}$$

Section 7.2

(14) $$\int \theta\sec^2(\theta)\,d\theta = \theta\tan\theta - \int \tan\theta\,d\theta = \underline{\underline{\theta\tan\theta - \ln|\sec\theta| + C}}$$

$w = \theta$ $\quad dv = \sec^2\theta\,d\theta$

$dw = d\theta$ $\quad v = \tan\theta$

(16) $$\int x\ln(x)\,dx = \ln(x)\cdot\frac{x^2}{2} - \int \frac{x^2}{2}\cdot\frac{1}{x}\,dx$$

$w = \ln(x)$ $\quad dv = x\,dx$

$dw = \frac{1}{x}\,dx$ $\quad v = \frac{x^2}{2}$

$$= \frac{x^2}{2}\ln(x) - \int \frac{x}{2}\,dx$$

$$= \underline{\underline{\frac{x^2}{2}\ln(x) - \frac{x^2}{4} + C}}$$

(18) $$\int \sin^{-1}(x)\,dx = \sin^{-1}(x)\cdot x - \int x\cdot\frac{1}{\sqrt{1-x^2}}\,dx$$

$w = \sin^{-1}(x)$ $\quad dv = dx$

$dw = \frac{1}{\sqrt{1-x^2}}\,dx$ $\quad v = x$

$$= \underline{\underline{x\sin^{-1}(x) + \sqrt{1-x^2} + C}}$$

$$\int \frac{x}{\sqrt{1-x^2}}\,dx = -\frac{1}{2}\int \theta^{-1/2}\,d\theta = -\theta^{1/2}+C = -\sqrt{1-x^2}+C$$

$\theta = 1-x^2$, $d\theta = -2x\,dx$

(24) $\int e^{3x}\cos(2x)\,dx = e^{3x}\cdot\frac{1}{2}\sin(2x) - \frac{3}{2}\int e^{3x}\sin(2x)\,dx$ $w = e^{3x}$, $dw = 3e^{x}dx$ $dv = \sin(2x)\,dx$, $v = -\frac{1}{2}\cos(2x)$

$w = e^{3x}$ $dv = \cos(2x)$

$dw = 3e^{3x}$ $v = \frac{1}{2}\sin(2x)$

$$= \frac{1}{2}e^{3x}\sin 2x - \frac{3}{2}\left[e^{3x}\cdot\left(-\frac{1}{2}\cos 2x\right) - \int -\frac{1}{2}\cos(2x)\cdot 3e^{x}\,dx\right]$$

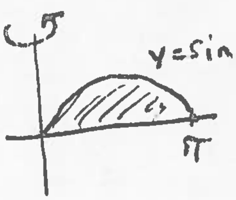
So
$$\int e^{3x}\cos(2x)\,dx = \frac{1}{2}e^{3x}\sin(2x) + \frac{3}{4}e^{3x}\cos(2x) - \frac{9}{4}\int e^{3x}\cos(2x)\,dx$$

$$\left(1+\frac{9}{4}\right)\int e^{3x}\cos(2x)\,dx = \frac{1}{2}e^{3x}\sin(2x) + \frac{3}{4}e^{3x}\cos(2x)$$

$$\int e^{3x}\cos(2x)\,dx = \frac{4}{13}\left(\frac{1}{2}e^{3x}\sin(2x) + \frac{3}{4}e^{3x}\cos(2x)\right) + C$$

$$= \underline{\frac{2}{13}e^{3x}\sin(2x) + \frac{3}{13}e^{3x}\cos(2x) + C}$$

(40)



$$V = \int_a^b 2\pi x f(x)\,dx = 2\pi\int_0^{\pi} x\sin(x)\,dx$$

$= 2\pi\left(-x\cos(x) + \sin(x)\Big|_0^{\pi}\right)$ [done in class]

$= 2\pi\left[(-\pi\cos(\pi) + \sin(\pi)) - (0\cdot\cos(0) + \sin(0))\right]$

$= 2\pi(\pi) = \underline{\underline{2\pi^2}}$

(46) $\int x^m \sin(ax)\,dx$

$w = x^m$ $dv = \sin(ax)\,dx$

$dw = m x^{m-1}\,dx$ $v = -\frac{1}{a}\cos(ax)$

So,
$$\int x^m \sin(ax) = x^m\cdot\left(-\frac{1}{a}\cos(ax)\right) - \int -\frac{1}{a}\cos(ax)\cdot m x^{m-1}\,dx$$

$$\underline{\int x^m \sin(ax) = -\frac{x^m}{a}\cos(ax) + \frac{m}{a}\int x^{m-1}\cos(ax)\,dx}$$