Math 131-03, Spring 2017 Sample Answers To Homework #8

Section 7.8

(6) $\int_0^\infty \frac{dx}{(x+1)^3} = \lim_{b\to\infty} \int_0^b \frac{dx}{(x+1)^3} = \lim_{b\to\infty}\left(\frac{-1}{2(x+1)^2}\Big|_0^b\right) = \lim_{b\to\infty}\left(\frac{-1}{2(b+1)^2}+\frac{1}{2}\right) = \underline{\underline{\frac{1}{2}}}$

(28) $\int_1^\infty \frac{\tan^{-1}(s)}{s^2+1}\,ds$ $\left[\int \frac{\tan^{-1}(s)}{s^2+1}\,ds = \int w\,dw = \frac{w^2}{2}+C = \frac{1}{2}(\tan^{-1}(s))^2+C\right]$

$(w=\tan^{-1}(s),\ dw=\frac{1}{s^2+1}ds)$

$\int_1^\infty \frac{\tan^{-1}(s)}{s^2+1}\,ds = \lim_{b\to\infty}\left(\frac{1}{2}(\tan^{-1}(s))^2\Big|_1^b\right) = \lim_{b\to\infty}\left(\frac{1}{2}(\tan^{-1}(b))^2-\frac{1}{2}(\tan^{-1}(1))^2\right)$

$= \underline{\underline{\frac{1}{2}\left(\frac{\pi}{2}\right)^2-\frac{1}{2}\left(\frac{\pi}{4}\right)^2}}$, since $\tan^{-1}(1)=\frac{\pi}{4}$ and $\lim_{b\to\infty}\tan^{-1}(b)=\frac{\pi}{2}$

(40) $\int_3^4 \frac{dz}{(z-3)^{3/2}} = \lim_{a\to3^+}\int_a^4 \frac{dz}{(z-3)^{3/2}} = \lim_{a\to3^+}\left(-\frac{2}{(z-3)^{1/2}}\Big|_a^4\right)$

$= \lim_{a\to3^+}\left(-\frac{2}{(4-3)^{1/2}}+\frac{2}{(a-3)^{1/2}}\right) = \underline{\underline{\infty}}$ (Diverges To ∞)

Section 7.9

(18) $\frac{dy}{dx} = 8e^{-4t}+1,\ y(0)=5$

$dy = (8e^{-4t}+1)\,dt$

$\int dy = \int 8e^{-4t}+1\,dt$

$y = -2e^{-4t}+t+C$

since $y(0)=5$, we have

$5 = -2e^0+0+C$

$5=-2+C$

$7=C$

So: $\underline{\underline{y=-2e^{-4t}+t+7}}$

(32) $\frac{dy}{dx} = y(x^2+1)$, where $y>0$

$\frac{1}{y}dy = (x^2+1)\,dx$

$\int\frac{1}{y}dy = \int x^2+1\,dx$

$\ln|y| = \frac{x^3}{3}+x+C$

But since $y>0$, we can write $\ln(y)$ instead of $\ln|y|$:

$\ln(y) = \frac{x^3}{3}+x+C$

$e^{\ln(y)} = e^{x^3/3+x+C}$

$\underline{\underline{y = e^{x^3/3+x+C}}}$

or $\underline{\underline{y = Ae^{x^3/3+x}}}$ $(A>0)$