① $\frac{d}{dx}\left(\int_0^{2x^3} e^{t^2}dt\right) = e^{(2x^3)^2}\cdot\frac{d}{dx}(2x^3) = e^{(2x^3)^2}\cdot 6x^2$

[By The Fundamental Theorem of calculus and The chain rule]

② Using a Left Riemann Sum, with 8 subintervals and $\Delta x = \frac{1}{2}$:

$1.3\times\frac{1}{2} + 1.6\times\frac{1}{2} + 1.85\times\frac{1}{2} + 2.12\times\frac{1}{2} + 2.4\times\frac{1}{2} + 2.6\times\frac{1}{2} + 2.77\times\frac{1}{2} + 3.0\times\frac{1}{2}$

③ a) $\int \frac{x^2+1}{x^3+3x}dx = \int \frac{1}{w}\cdot\frac{1}{3}dw = \frac{1}{3}\ln|w| + C = \underline{\frac{1}{3}\ln|x^3+3x| + C}$

$w = x^3+3x$, $dw = (3x^2+3)dx = 3(x^2+1)dx$

b) $\int x^2 + xe^{2x}dx = \int x^2 dx + \int xe^{2x}dx = \underline{\frac{x^3}{3} + \left(\frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x}\right) + C}$

(For $\int xe^{2x}dx = x\cdot\frac{1}{2}e^{2x} - \int\frac{1}{2}e^{2x}dx = \frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x} + C$

$w = x$ $dv = e^{2x}dx$

$dw = dx$ $v = \frac{1}{2}e^{2x}$)

c) $\frac{5x-3}{2x(x-1)} = \frac{A}{2x} + \frac{B}{x-1}$

$5x-3 = A(x-1) + B\cdot 2x$

$x=1 \Rightarrow 2 = B\cdot 2 \Rightarrow B = 1$

$x=0 \Rightarrow -3 = A\cdot(-1) \Rightarrow A = 3$

partial fractions gives $\frac{5x-3}{2x(x-1)} = \frac{3}{2x} + \frac{1}{x-1}$

$\int\frac{5x-3}{2x(x-1)}dx = \int\frac{3}{2x} + \frac{1}{x-1}dx$

$= \underline{\frac{3}{2}\ln|x| + \ln|x-1| + C}$

(d) $\int \ln(x+1)dx$

$w = \ln(x+1)$ $dv = dx$

$dw = \frac{1}{x+1}dx$ $v = x$

$\int\ln(x+1)dx = x\ln(x+1) - \int x\cdot\frac{1}{x+1}dx$

$= x\ln(x+1) - \int\frac{x+1-1}{x+1}dx$

$= x\ln(x+1) - \int 1 - \frac{1}{x+1}dx$

$= \underline{x\ln(x+1) - x + \ln(x+1) + C}$

④ 

Disk method

$\int_0^2 \pi R^2 dx$

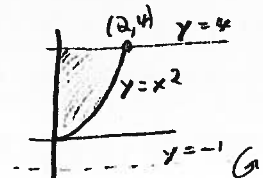
$= \underline{\int_0^2 \pi(e^x)^2 dx}$

⑤ 

Shell Method

$\int_0^{\pi/2} 2\pi r h\, dx$

$= \underline{\int_0^{\pi/2} 2\pi x f(x) dx}$

⑥ 

washer method

$\int_0^2 \pi(R^2 - r^2)dx$

$= \underline{\int_0^1 \pi\left[(4+1)^2 - (x^2+1)^2\right]dx}$

(7) $y\,dy = \frac{1}{1+x^2}dx \Rightarrow \int y\,dy = \int \frac{dx}{1+x^2} \Rightarrow \frac{y^2}{2} = \tan^{-1}(x) + C$

$\Rightarrow y = \pm\sqrt{2(\tan^{-1}(x)+C)}$. Plugging in $y(0)=1$ gives

$1 = \pm\sqrt{2(\tan^{-1}(0)+C)} \Rightarrow$ The sign is positive and $C = \frac{1}{2}$ (since $\tan^{-1}(0)=0$).

So The solution is $y = \sqrt{2(\tan^{-1}(x)+\frac{1}{2})}$ or $\underline{y = \sqrt{2\tan^{-1}(x)+1}}$

(8) a) $\frac{n^2}{n^4+1} < \frac{n^2}{n^4} = \frac{1}{n^2}$, Since $\sum_{n=1}^{\infty}\frac{1}{n^2}$ is a convergent p-series,

$\sum_{n=1}^{\infty}\frac{n^2}{n^4+1}$ also <u>converges</u> by The comparison Test.

b) <u>diverges</u> by limit comparison Test, comparing to divergent p-series $\sum_{n=1}^{\infty}\frac{1}{n}$:

$\lim_{n\to\infty} \frac{n^2}{n^3+1}\Big/\frac{1}{n} = \lim_{n\to\infty}\frac{n^3}{n^3+1} = 1$. (The Test applies since This limit is between 0 and ∞.)

c) <u>diverges</u> by Divergence Test, since $\lim_{n\to\infty}\frac{n^2}{n^2+1} = 1 \neq 0$

d) <u>converges</u> by Alternating Series Test since The series $\left\{\frac{1}{\sqrt{k}}\right\}_{k=1}^{\infty}$ is decreasing and $\lim_{n\to\infty}\frac{1}{\sqrt{k}} = 0$.

(9) Apply The ratio Test: $\lim_{n\to\infty}\frac{|a_{n+1}|}{|a_n|} = \lim_{n\to\infty}\left|\frac{(n+1)^2x^{n+1}}{5^{n+1}}\right|\Big/\frac{n^2x^n}{5^n}$

$= \lim_{n\to 0}\left|\frac{(n+1)^2x^{n+1}5^n}{5^{n+1}n^2x^n}\right| = \lim_{n\to\infty}\frac{(n+1)^2}{n^2}\cdot|x|\cdot\frac{1}{5} = 1\cdot|x|\cdot\frac{1}{5} = \frac{|x|}{5}$.

The series converges if $\frac{|x|}{5}<1$, i.e $|x|<5$, so radius of convergence is $\underline{\underline{5}}$. At The endpoints, ± 5, The series becomes:

$x=-5$: $\sum_{n=1}^{\infty}\frac{n^2(-5)^n}{5^n} = \sum_{n=1}^{\infty}(-1)^n n^2$, $x=5$: $\sum_{n=1}^{\infty}\frac{n^2\cdot 5^n}{5^n} = \sum_{n=1}^{\infty}n^2$

Both of These series diverge (by Divergence Test), so The interval of convergence is $\underline{\underline{(-5,5)}}$.

(10) If one of $a_n, \frac{1}{a_n}$ goes To zero, The other goes To infinity.
A series can only converge if its Terms go To zero, so
$\sum_{k=1}^{\infty}a_k$ and $\sum_{k=1}^{\infty}\frac{1}{a_k}$ <u>cannot</u> both converge. It <u>is possible</u>
for both series To diverge, for example: $\sum_{k=1}^{\infty}\frac{k+1}{k}$ and $\sum_{k=1}^{\infty}\frac{k}{k+1}$