Lab report is due at the start of next week's lab.

1. In class we discussed why the identities $\int_a^b cf(x)\,dx = c\cdot\int_a^b f(x)\,dx$ and $\int_a^b f(x)+g(x)\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$ make sense in terms of Riemann sums. We also stated the identity $\int_a^c f(x)\,dx = \int_a^b f(x)\,dx + \int_b^c f(x)\,dx$, but we only justified it by looking at the areas corresponding to the integrals. Integrals are not *defined* as areas; they are defined as limits of Riemann sums. Give a convincing explanation using Riemann sums that the identity

$$\int_a^c f(x)\,dx = \int_a^b f(x)\,dx + \int_b^c f(x)\,dx \qquad \text{where } a < b < c$$

is true. You should answer with a short essay that includes **pictures** to illustrate your ideas. Start by considering a Riemann sum for the integral on the left. You are not being asked to give a formal proof, just a convincing explanation.

2. We have defined the definite integral of a function on a closed interval $[a, b]$. In order to have a Riemann sum for an interval, the length of the interval must be finite. So, the numbers a and b have to be finite. But of course, as usual, we can use limits to investigate what happens as some quantity approaches infinity. Here, for example, we can consider $\lim\limits_{b\to\infty} \int_1^b f(x)\,dx$. Apply this idea to each of the following cases, and compute the value of each limit:

a) $\displaystyle\lim_{b\to\infty}\left(\int_1^b \frac{1}{x^2}\,dx\right)$ **b)** $\displaystyle\lim_{b\to\infty}\left(\int_1^b \frac{1}{\sqrt{x}}\,dx\right)$ **c)** $\displaystyle\lim_{b\to\infty}\left(\int_1^b \frac{1}{x}\,dx\right)$

d) Explain what these limits say in terms of **areas**. What area is represented by each limit, and is that area infinite?

3. Just yesterday, we covered the *method of substitution* for evaluating indefinite integrals. Find the indefinite integral

$$\int \sin(x)\cos(x)\,dx$$

in **two** different ways, first by using the substitution $u = \cos(x)$ and then by using the substitution $u = \sin(x)$. Then explain why the two answers are actually the same.

4. This problem is another exercise in computing indefinite integrals by substitution. We know that $\int x^{11}\,dx = \frac{1}{12}x^{12} + C$. Compute $\int x^{11}\,dx$ using each of the following substitutions, and check that the answer is correct in each case: $u = x^6$, $u = x^3$, and $u = x^2$.

5. Using the FToC, we can compute $\int_1^e \frac{1}{x}\,dx = \ln(x)\Big|_1^e = \ln(e) - \ln(1) = 1 - 0 = 1$. But how do we know the value of e? Use a right Riemann sum with 8 subintervals to show that $\int_0^3 \frac{1}{x}\,dx > 1$. Explain your answer and explain why this shows that $e < 3$. To illustrate your answer, draw pictures that show the areas represented by the two integrals and by the Riemann sum. (Similar computations using larger number of subintervals can give better upper bounds for e. And similar computations using left Riemann sums can give lower bounds. But this is not the best way to estimate e.)