This lab is due at the start of next week's lab.

1. Suppose that f is a function that is continuous on $[0,\infty]$ and that $\lim_{x\to\infty} f(x) = 1$. Consider the **average value** of $f(x)$ on the interval $[0,b]$. What do you expect to happen to this average value as $b \to \infty$? That is, what is $\lim_{b\to\infty}\left(\frac{1}{b}\int_0^b f(x)\,dx\right)$? Why? Explain your reasoning. Draw a picture.

2. Consider the function $f(x) = \begin{cases} x, & 0 \le x < 2 \\ 3, & 2 \le x \le 5 \end{cases}$. This function has a jump discontinuity at $x = 2$. (Recall that this means the left and right limits, $\lim_{x\to 2-} f(x)$ and $\lim_{x\to 2+} f(x)$, both exist but are different.) Nevertheless, f is integrable on the interval $[0,5]$, and we can define the area function $A(x) = \int_0^x f(t)\,dt$.

a) Draw the graph of f on the interval $[0,5]$.

b) Find an explicit formula for the area function $A(x) = \int_0^x f(t)\,dt$ on the interval $[0,5]$. It will be similar to the formula for $f(x)$; that is, it will be a split function. (Show your work!)

c) The Fundamental Theorem of Calculus implies that $A(x)$ is differentiable on any interval on which $f(x)$ is continuous, but it doesn't say anything about what happens at $x = 2$, where $f(x)$ has a discontinuity. So, what does happen with $A(x)$ at $x = 2$? Is A differentiable at $x = 2$? Is A continuous at $x = 2$? Why? [Look at what happens as you approach $x = 2$ from the left and from the right.]

d) Try to come up with a hypothesis about what happens to any area function $A(x) = \int_a^x f(t)\,dt$ at a value of x where f has a jump discontinuity. State your hypothesis clearly, and try to specify as much as you can about the behavior of A at the point where f is discontinuous.

3. Sometimes, an indefinite integral can be found using a substitution that is less than obvious. Compute the following integrals by using the suggested substitution.

a) $\displaystyle\int \frac{x^3}{\sqrt{2x^2+1}}\,dx$, using $u = 2x^2+1$. (Hint: Write x^2 in terms of u.)

b) $\displaystyle\int \frac{1}{\sqrt{x}\,(1+x)}\,dx$, using $u = \sqrt{x}$. (Hint: Write x in terms of u.)

c) $\displaystyle\int \sin^3(x)\,dx$, using $u = \cos(x)$. (Hint: Write $\sin^2(x) = 1 - \cos^2(x)$.)

Turn over for Problem 4

4. In each of the following problems, fill in the box with a *non-zero* function that will make the integral doable. Then find the resulting integral.

a) $\displaystyle\int \boxed{} \cdot \sin(x^2 + 3x + 7)\, dx$

b) $\displaystyle\int \sqrt{x} \cdot e^{\boxed{}}\, dx$

c) $\displaystyle\int \sqrt[3]{\boxed{}} \cdot (3e^{3x} + 3e^{-3x})\, dx$

If you have extra time in the lab,
consider working on this homework, which is
due next Tuesday along with the lab:

Section 6.1, # 8, 30, 34, 40, 60