This lab is review for tomorrow's test. It will not be collected. Answers will be available before the end of the lab. Problems 1 to 10 are the complete first test from a Spring 2011 Calculus II. That test did not cover Section 6.1. The averages on that test was 87%. You can expect our test to be a little more difficult.

Problems 11 and beyond are some additional review exercises, including some on Section 6.1. But note that this lab is not meant to cover every single topic or type of problem that might be on the test.

1. Compute the value of the definite integral $\int_1^3 x^2 - 1\,dx$ and sketch the area that is represented by this integral.

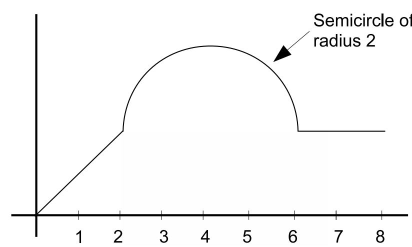
2. Compute the following indefinite integrals, using substitution:

a) $\int 3x^2\cos(x^3)\,dx$ **b)** $\int \frac{e^x}{(3e^x+7)^2}\,dx$

3. Use geometry to find $\int_0^8 f(x)\,dx$, where f is the following function, whose graph is as shown on the right:

$$f(x) = \begin{cases} x & \text{if } 0 \le x < 2 \\ 2 + \sqrt{4-(x-4)^2} & \text{if } 2 \le x \le 6 \\ 2 & \text{if } x > 6 \end{cases}$$

Show your work!



4. Consider the definite integral $\int_0^3 x\sqrt{100-x^4}\,dx$. Apply the substitution $u = x^2$ to this integral to completely re-write the integral in terms of u, including the limits of integration. You are only being asked to make the substitution; **do not** try to evaluate the resulting integral.

5. Use the substitution $u = x^3$ to compute the indefinite integral $\int \frac{x^2}{1+x^6}\,dx$

6. A *Riemann Sum* can be written in the form $\sum_{k=1}^{n} f(x_k^*)\Delta x$. Riemann sums were introduced as a way of approximating the area under the graph of a function $y = f(x)$ (assuming $f(x) \ge 0$). Exactly what area does the Riemann sum represent? *Why* is the Riemann sum an approximation for the area under the curve? Answer with a short essay, and draw a picture that shows a graph $y = f(x)$ and the area represented by a Riemann sum for $f(x)$.

7. Use a right Riemann sum with 4 subintervals to estimate the area under the graph $y = x^3$ on the interval $[1, 3]$. (Your answer can be left in the form of a sum of numbers; you don't have to add up the numbers.)

8. Find the *average value* of the function $f(x) = \dfrac{1}{x}$ on the interval $[1, e]$.

9. Suppose that f is a continuous function on the interval $[a, b]$.

a) Suppose that $\displaystyle\int_a^b f(x)\,dx = 0$. Does it have to be true that $f(x) = 0$ for all x in $[a, b]$? Why or why not?

b) Suppose that $\displaystyle\int_a^b |f(x)|\,dx = 0$. Does it have to be true that $f(x) = 0$ for all x in $[a, b]$? Why or why not?

10. The Fundamental Theorem of Calculus has two parts. Pick *one* of the two parts and state that part of the theorem. Then explain briefly why it is *important*. (For example, how is it used to solve problems that would be difficult to solve without the fundamental theorem?)

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11. A point moves along a line from time $t = 0$ to time $t = 10$ with velocity given by $v(t) = 64 - t^2$. If the initial position of the point is $s(0) = 20$, find its position at time $t = 10$. Also find the distance that the point travels on the time interval $0 \le t \le 10$.

12. Water drips into a 500 cubic centimeter cup. The amount of water in the cup at time t is given by $Q(t)$. The cup is empty at time $t = 0$, and the rate at which water drips into the cup is given by $Q'(t) = t + 15$ cubic centimeters per second. At what time will the cup be full?

13. The values of a function $f(x)$ are shown in the following table for certain values of x. Using this data, write a Riemann sum for $f(x)$ on the interval $[1, 4]$, using six sub-intervals. (There is more than one possible answer.)

x	1.0	1.5	2.0	2.5	3.0	3.5	4.0
$f(x)$	1.5	1.0	1.2	1.7	2.9	3.3	3.6

14. Find the area of the region bounded by the x-axis, the lines $x = 1$ and $x = 3$, and the graph of the function $y = x^2 e^{x^3}$.

15. Find the derivative: $\displaystyle\frac{d}{dx}\left(\int_0^{3x} \sqrt{z^3 + 1}\,dz\right)$

16. Write $\displaystyle\int_0^3 2x + 1\,dx$ as a limit of right Riemann sums. (You are not being asked to evaluate the limit.)

17. Some more integrals to practice on:

a) $\displaystyle\int \cos(6x)\,dx$ **b)** $\displaystyle\int (x+1)\sqrt{x^2 + 2x + 3}\,dx$ **c)** $\displaystyle\int 4t^5(1 + \sqrt{t})\,dt$

d) $\displaystyle\int \frac{\sec^2(x)}{1 + \tan(x)}\,dx$ **e)** $\displaystyle\int_1^4 4x^3 - 6x\,dx$ **f)** $\displaystyle\int_0^2 x\sqrt{4 - x^2}\,dx$