

The first test for this course will be given in class on Wednesday, February 15. The test will cover all of Chapter 5 plus Section 6.1. (But note that we did very little from Section 5.4). You can expect any test that I give to include some “short essay” questions that ask you to define something, discuss something, explain something, state a theorem, and so on. There might be one longer essay that requires more depth of understanding. Other than that, most of the questions will be similar to problems you have seen on the homework or Webwork or as examples in class.

You will not need a calculator for the test. A basic non-graphing calculator will be provided to you, and you will be permitted to use only the calculator that is provided. However, you will probably not need a calculator at all. Scrap paper will also be provided. All you need is a pencil or pen.

Here are some terms and ideas that you should be familiar with for the test:

Riemann sum

Using Riemann sums to approximate areas

Left Riemann sum, right Riemann sum, and midpoint Riemann sum

Geometric meaning of a Riemann sum (as a sum of areas of rectangles)

Summation notation

Summation notation for a Riemann sum: $\sum_{k=1}^n f(x_k^*) \Delta x$

$\sum_{k=1}^n k = \frac{n(n+1)}{2}$ and similar sum formulas **will be given to you, if needed**

Definite integral, $\int_a^b f(x) dx$

The “ x ” in a definite integral is a dummy variable; $\int_a^b f(x) dx = \int_a^b f(t) dt$

Relationship between definite integral and area; how to deal with area below the x -axis

Computing an exact integral or area using a limit of Riemann sums: $\lim_{n \rightarrow \infty} \left(\sum_{k=1}^n f(x_k^*) \Delta x \right)$

Properties of definite integrals:

$$\int_a^b k \cdot f(x) dx = k \cdot \int_a^b f(x) dx, \text{ for a constant } k$$

$$\int_a^b f(x) + g(x) dx = \int_a^b f(x) dx + \int_a^b g(x) dx$$

$$\int_a^b f(x) dx = \int_a^c f(x) dx + \int_c^b f(x) dx$$

There is no product rule, no quotient rule, and no chain rule for integrals!

Evaluating definite integrals geometrically, using areas of simple shapes

Fundamental Theorem of Calculus:

Part 1: If f is continuous on $[a, b]$, and if F is defined as $F(x) = \int_a^x f(t) dt$ for x in $[a, b]$, then F is an antiderivative of f on $[a, b]$ (that is, $F'(x) = f(x)$).

Part 2: If f is continuous on $[a, b]$, and if F is any antiderivative of f on $[a, b]$, then $\int_a^b f(x) dx = F(b) - F(a)$.

The Fundamental Theorem, Part 1, says that every continuous function has an antiderivative, even if it can't be written in terms of the elementary functions. The Fundamental Theorem, Part 2, is the basic tool for actually evaluating definite integrals (at least for functions whose antiderivatives are known.)

$$\frac{d}{dx} \int_a^x f(t) dt = f(x), \text{ for a continuous function } f$$

The notation $F(x)|_a^b$

Average value of an integrable function on an interval: $\frac{1}{b-a} \int_a^b f(x) dx$

Using substitution to compute indefinite integrals (**this is a major topic!**)

Substitution in definite integrals

Net change in a quantity $Q(t)$ for $a \leq t \leq b$ is given by $\int_a^b Q'(t) dt$

$$Q(t) = Q(0) + \int_0^t Q'(x) dx$$

For motion along a line, with position given by $s(t)$ for $a \leq t \leq b$:

displacement = $\int_a^b v(t) dt$, where $v(t)$ is the velocity

distance traveled = $\int_a^b |v(t)| dt$, where $|v(t)|$ gives the speed

$v(t) = v(0) + \int_0^t a(x) dx$, where $a(t)$ is the acceleration

$s(t) = s(0) + \int_0^t v(x) dx$, where $v(t)$ is the velocity

You are expected to know certain things from Calculus I. Most important, you should have memorized the basic derivative and antiderivative formulas. Here are the antiderivative formulas that you are expected to know:

$$\begin{aligned} \int x^k dx &= \frac{1}{k+1} x^{k+1} + C, \text{ if } k \text{ is a constant, not equal to } -1 \\ \int \frac{1}{x} dx &= \ln|x| + C & \int e^x dx &= e^x + C \\ \int \sin(x) dx &= -\cos(x) + C & \int \cos(x) dx &= \sin(x) + C \\ \int \sec^2(x) dx &= \tan(x) + C & \int \sec(x) \tan(x) dx &= \sec(x) + C \\ \int \frac{1}{1+x^2} dx &= \tan^{-1}(x) + C & \int \frac{1}{\sqrt{1-x^2}} dx &= \sin^{-1}(x) + C \end{aligned}$$