This homework is due by ***noon*** *on Thursday, October 1,*
There is a ***test*** *on Friday, October 2.*
Because of the test, this homework will not be accepted late,
and there will be no rewrites.
Solutions for Homeworks 4 an 5 will be published at noon on October 1.

Problem 1. Decide whether each set of vectors is a basis for $\mathbb{R}^3$. Give a reason for your answer. In some cases, the reason can be very short. In other cases, a calculation is required.

(a) $\left\{ \begin{pmatrix} 1 \\ 3 \\ 7 \end{pmatrix}, \begin{pmatrix} 2 \\ 0 \\ 4 \end{pmatrix} \right\}$

(b) $\left\{ \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 8 \\ 5 \\ 1 \end{pmatrix}, \begin{pmatrix} 4 \\ 7 \\ 5 \end{pmatrix} \right\}$

(c) $\left\{ \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}, \begin{pmatrix} 2 \\ 6 \\ 4 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \right\}$

(d) $\left\{ \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}, \begin{pmatrix} 3 \\ 4 \\ 5 \end{pmatrix} \right\}$

Problem 2. Suppose that $(V, +, \cdot)$ is a vector space and $\mathcal{U}$ is a basis of V, where $\mathcal{U} = \langle \vec{\beta}_1, \vec{\beta}_2, \ldots, \vec{\beta}_n \rangle$. Let $\mathcal{D} = \langle \vec{\beta}_n, \vec{\beta}_{n-1}, \ldots, \vec{\beta}_1 \rangle$. (Then $\mathcal{D}$ is also a basis of V. It is a different basis, since these bases are ordered.) For a vector $\vec{v} \in V$, how does $\mathrm{Rep}_{\mathcal{U}}(\vec{v})$ compare to $\mathrm{Rep}_{\mathcal{D}}(\vec{v})$? Justify your answer.

Problem 3. The sequence $\mathcal{B} = \left\langle \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \right\rangle$ is a basis for $\mathbb{R}^3$. Let $\vec{v} = \begin{pmatrix} 5 \\ 7 \\ 3 \end{pmatrix}$. Find $\mathrm{Rep}_{\mathcal{B}}(\vec{v})$. (You do **not** need to prove that $\mathcal{B}$ is a basis.)

Problem 4. Using the basis, $\mathcal{B}$, from problem 3, suppose that $\mathrm{Rep}_{\mathcal{B}}(\vec{v}) = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$. Find $\vec{v}$.

Problem 5. Let $\mathscr{P}_3$ be the vector space of polynomials of degree less than or equal to 3. Let $\mathcal{B}$ be the basis of $\mathscr{P}_3$ given by

$$\mathcal{B} = \langle 1 + x, x - x^2, 1 + x^3, 2x - x^2 + x^3 \rangle$$

Find $\mathrm{Rep}_{\mathcal{B}}(3 - 4x + 4x^2 + x^3)$. (You do **not** have to prove that $\mathcal{B}$ is a basis.)