This homework is due on Monday, May 4.

1. Some exercises about complex numbers...

a) Let a,b,c, and d be real numbers with $d \neq 0$. Write $\frac{a+bi}{c+di}$ in the form $\alpha + i\beta$ where α and β are real numbers. Hint: Multiply by $\frac{c-di}{c-di}$.

b) Let $z = a + bi$ be a complex number, where a and b are real. The norm of z is defined by $|z| = |a+bi| = \sqrt{a^2+b^2}$. Show that for two complex nubmers z and w, $|zw| = |z| \cdot |w|$.

c) Use the fact that $e^{i\theta} = \cos(\theta) + i\sin(\theta)$ and the fact that $e^{i(\theta+\varphi)} = e^{i\theta} \cdot e^{i\varphi}$ to derive the angle sum formulas for sine and cosine.

2. Let A be the complex matrix $A = \begin{pmatrix} -1 & i & 0 \\ i & -2 & 1-i \\ 1 & 3-i & 1+i \end{pmatrix}$. Find the inverse matrix A^{-1} (by forming the augmented matrix $(A|I_n)$ and putting it into reduced row echelon form).

3. Find all (complex) eigenvalues for the following matrices. (At least one of these is really easy.)

a) $\begin{pmatrix} 1 & 5 \\ -2 & 3 \end{pmatrix}$ **b)** $\begin{pmatrix} 1 & 2 & 3 \\ 0 & i & 1-i \\ 0 & 0 & 1+i \end{pmatrix}$ **c)** $\begin{pmatrix} 3 & 0 & 0 \\ 1 & 5 & -5 \\ 0 & 3 & -1 \end{pmatrix}$

4. Suppose that λ is an eigenvalue for the matrix A. Show that λ^2 is an eigenvalue for the matrix $A^2 = AA$. Now suppose that A is non-singular; in that case, can you say anything about eigenvalues of the inverse matrix, A^{-1}?

5. Suppose that $h\colon \mathbb{C}^2 \to \mathbb{C}^2$ is a homomorphism, i is an eigenvalue of h with eigenvector $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$, and $-i$ is an eigenvalue of h with eigenvector $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$. Find the matrix of h in the standard basis.

6. Let $A = \begin{pmatrix} 1 & a & 0 \\ 0 & 1 & a \\ 0 & 0 & 1 \end{pmatrix}$, and assume that $a \neq 0$. Show that A is not diagonalizable. (You can do this by showing that $\mathbb{C}^3$ does not have a basis of eigenvectors of A.)

7. Let $\mathscr{D}$ be the vector space of differentiable functions from $\mathbb{R}$ to $\mathbb{R}$. That is, $\mathscr{D}$ is the set $\{f\colon \mathbb{R} \to \mathbb{R} \,|\, f'(x) \text{ exists for all } x\}$, with the usual addition and scalar multiplication for real-valued functions. Let $\partial\colon \mathscr{D} \to \mathscr{D}$ be the derivative function, $\partial(f) = f'$. Show that every real number λ is an eigenvalue for ∂, and find an eigenvector for eigenvalue λ. (You need to remember a fact from calculus.)