This homework on Sections I.3, II.1, and II.2 is due in class on Wednesday, February 5.

1. Determine whether each matrix is singular or non-singular. As part of the justification for you answers, you will have to explain in words why you can answer the question by putting the matrix into echelon form.

a) $\begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}$ **b)** $\begin{pmatrix} 1 & 3 & -1 \\ 2 & 0 & 3 \\ -1 & 2 & 1 \end{pmatrix}$ **c)** $\begin{pmatrix} -1 & 0 & 1 & 0 \\ 3 & 2 & -2 & 4 \\ 2 & 1 & 0 & -1 \\ 1 & 1 & 0 & 1 \end{pmatrix}$

2. Let $\vec{u}$ and $\vec{v}$ be vectors in $\mathbb{R}^n$, and let r and s be real numbers. By writing out $\vec{u}$ and $\vec{v}$ in coordinates, verify the following identities:

a) $r(\vec{v} + \vec{u}) = r\vec{v} + r\vec{u}$

b) $(r + s)\vec{v} = r\vec{v} + s\vec{u}$

c) $(r\vec{v}) \cdot \vec{u} = r(\vec{v} \cdot \vec{u})$

3. Let $\vec{u}$ and $\vec{v}$ be vectors in $\mathbb{R}^n$, and let a and b be non-zero real numbers. Verify that the angle between $\vec{v}$ and $\vec{u}$ is the same as the angle between $a\vec{v}$ and $b\vec{u}$.

4. Let $\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_{n-1}$ be $n - 1$ vectors in $\mathbb{R}^n$. Prove that there is a non-zero vector $\vec{x}$ in $\mathbb{R}^n$ that is orthogonal to $\vec{v}_i$ for $i = 1, 2, \ldots, n - 1$. (Hint: Think about linear equations!) Now, find n vectors $\vec{u}_1, \vec{u}_2, \ldots, \vec{u}_n$ such that the $\vec{0}$ is the only vector in $\mathbb{R}^n$ that is orthogonal to $\vec{u}_i$ for $i = 1, 2, \ldots, n$. (Hint: Don't look for a complicated example!)

5. A plane in $\mathbb{R}^3$ can be given by an equation $Ax + By + Cz = D$ where A, B, C, and D are constants and A, B, and C are not all zero.

Suppose two planes are given by equations $A_1x+B_1y+C_1z = D_1$ and $A_2x+B_2y+C_2z = D_2$. The intersection of the two planes can be empty, or it can be a line, or the planes could be identical. How can the correct possibility be determined from the constants in the equations? Explain! (Hint: Think in terms of solving linear systems.)