*This homework on Chapter 1, Sections III.1 and III.2 and Chapter 1, Section I.1 is due in class on Friday, February 14).*

**1.** Produce three other matrices that are row equivalent to the following matrix. State what you did to get the new matrices.

$$\begin{pmatrix} 2 & 4 & -1 \\ 3 & 1 & 0 \\ -5 & 2 & 7 \end{pmatrix}$$

If you understand what row equivalence means, this exercise is very easy!

**2.** Apply row operations to put the following matrix into reduced row echelon form. Show the full sequence of operations that you apply

$$\begin{pmatrix} 1 & 0 & -2 & 0 & 2 \\ 0 & 1 & 3 & 2 & 7 \\ 1 & 2 & 4 & 2 & 8 \\ -2 & 2 & 10 & 2 & 2 \end{pmatrix}$$

(Note that you could check your final matrix at wolframalpha.com.)

**3.** Assuming that the matrix in the previous problem represents a homogeneous system of linear equations, use the reduced row echelon form of the matrix to find the solution set of that system.

**4.** Let $V$ be a subset of $\mathbb{R}^2$ that is a vector space using the addition and scalar multiplication operations from $\mathbb{R}^2$. Suppose that $\binom{1}{0} \in V$ and $\binom{0}{1} \in V$. Prove that $V = \mathbb{R}^2$. (This is easy!)

**5.** Remember that to show that a set is **not** a vector space, you only need to find one vector space property that fails.

- **a)** Let $S$ be the subset of $\mathbb{R}^2$ defined as $S = \{(x, y) \,|\, x + y = 1\}$. Show that $S$ is **not** a vector space, using the addition and scalar multiplication operations from $\mathbb{R}^2$.
- **b)** Let $T$ be the subset of $\mathbb{R}^2$ defined as $T = \{(x, y) \,|\, x + y > 0\}$. Show that $T$ is **not** a vector space, using the addition and scalar multiplication operations from $\mathbb{R}^2$.

**6.** We already know that the set $\mathscr{C}$ of continuous real-valued functions of one real variable is a vector space. Any subset of that space will automatically satisfy some of the vector space properties, such as all of the commutative, associative, and distributive laws. So, when you are trying to prove that a subset of $\mathscr{C}$ is a vector space, you don't have to check all of the vector space properties. This problem will require you to use some facts from calculus.

- **a)** Let $\mathscr{A} = \{f \in \mathscr{C} \,|\, \lim_{x\to\infty} f(x) = 0\}$. Show that $\mathscr{A}$ is a vector space, using the addition and scalar multiplication operations from $\mathscr{C}$.
- **b)** Let $\mathscr{B} = \{f \in \mathscr{C} \,|\, \int_0^1 f(x)\,dx = 0\}$. Show that $\mathscr{B}$ is a vector space, using the addition and scalar multiplication operations from $\mathscr{C}$.