This relatively homework on Chapter 2, Sections I.2 and II.1 is due in class on Wednesday, February 19. And remember that the first test is coming up on Friday, February 21. The test will cover Chapter 1 and Chapter 2, Sections I and II.

1. The following questions about spans can be answered by solving linear systems of equations.

a) Is the vector $\begin{pmatrix}1\\2\\3\end{pmatrix}$ in the span of the set $T = \left\{ \begin{pmatrix}-1\\0\\1\end{pmatrix}, \begin{pmatrix}1\\1\\1\end{pmatrix} \right\}$ in $\mathbb{R}^3$?

b) Is the vector $\begin{pmatrix}1\\1\\1\end{pmatrix}$ in the span of the set $T = \left\{ \begin{pmatrix}2\\-1\\1\end{pmatrix}, \begin{pmatrix}0\\1\\4\end{pmatrix}, \begin{pmatrix}-2\\2\\3\end{pmatrix} \right\}$ in $\mathbb{R}^3$?

2. Let T be the subset $T = \left\{ \begin{pmatrix}1\\0\\0\end{pmatrix}, \begin{pmatrix}1\\1\\0\end{pmatrix}, \begin{pmatrix}1\\1\\1\end{pmatrix} \right\}$ of $\mathbb{R}^3$. Show that $[T]$, the span of T, is all of $\mathbb{R}^3$. (You can approach this question by looking at solving a system of equations with constant terms that are unknowns.)

3. Let $\mathscr{P}$ be the (infinte-dimensional) vector space of all polynomials. Let T be the subset of $\mathscr{P}$ given by $\{p_0(x), p_1(x), p_2(x), \dots\}$, where $p_0(x) = 1$, $p_1(x) = 1 + x$, $p_2(x) = 1 + x + x^2$, $p_3(x) = 1+x+x^2+x^3$, and, more generally, $p_n(x) = 1+x+x^2+\cdots+x^n$. Show that $[T]$, the span of T, is all of $\mathscr{P}$. (You just need to check that any polynomial $q(x) = a_0+a_1x+a_2x^2+\cdots+a_kx^k$ can be written as a linear combination of some elements of T.)

4. Show that the following vectors in $\mathbb{R}^3$ are linearly dependent: $\begin{pmatrix}2\\-1\\1\end{pmatrix}, \begin{pmatrix}0\\1\\4\end{pmatrix}, \begin{pmatrix}-2\\2\\3\end{pmatrix}$.

5. Let $\mathscr{P}_3$ be the vector space of all polynomials of degree less that or equal to 3. Show that the following vectors in $\mathscr{P}_3$ are linearly independent: $1 - 2x$, $3x - 2x^2 + x^3$, $4 + x^2$.