This homework is due on Monday, March 1. It covers primarily Chapter 2, sections III.1 and III.2.

1. As stated in the course syllabus, you are required to do a small project and presentation that counts for 5% of your grade for the course. A list of possible topics was handed out on the first day of class. That list is now also available through a link on the course web page, *math.hws.edu/eck/math204*.

As part of this week's homework, you should pick three possible topics for your project. Your topics will probably come from the list that was provided; you can include other topics, but if you want to include a topic that is not on the list, you should discuss that topic idea with me before including it. You should rank your selected topics in order of preference. Please be clear about what each topic is! After I have everyone's selections, I will try to assign a topic to each person in the class. Wherever two or more people ask for the same topic, I will decide who gets that topic by a random process such as a coin toss.

2. Decide whether each set of vectors is a basis for $\mathbb{R}^3$. Give a reason for your answer. In some cases, the reason can be very short. In other cases, a calculation is required.

a) $\left\{ \begin{pmatrix}1\\3\\7\end{pmatrix}, \begin{pmatrix}2\\0\\4\end{pmatrix} \right\}$ **b)** $\left\{ \begin{pmatrix}1\\2\\3\end{pmatrix}, \begin{pmatrix}1\\1\\0\end{pmatrix}, \begin{pmatrix}8\\5\\1\end{pmatrix}, \begin{pmatrix}4\\7\\5\end{pmatrix} \right\}$

c) $\left\{ \begin{pmatrix}1\\3\\2\end{pmatrix}, \begin{pmatrix}2\\6\\4\end{pmatrix}, \begin{pmatrix}1\\0\\1\end{pmatrix} \right\}$ **d)** $\left\{ \begin{pmatrix}1\\2\\0\end{pmatrix}, \begin{pmatrix}2\\1\\3\end{pmatrix}, \begin{pmatrix}3\\4\\5\end{pmatrix} \right\}$

3. The set $B = \left\{ \begin{pmatrix}1\\0\\0\end{pmatrix}, \begin{pmatrix}1\\1\\0\end{pmatrix}, \begin{pmatrix}1\\1\\1\end{pmatrix} \right\}$ is a basis for $\mathbb{R}^3$. Let $\vec{v} = \begin{pmatrix}3\\7\\2\end{pmatrix}$, and find $\text{Rep}_B(\vec{v})$.

4. Using the same basis, B, from the previous problem, suppose that $\text{Rep}_B(\vec{v}) = \begin{pmatrix}3\\2\\5\end{pmatrix}$. Find $\vec{v}$.

5. Suppose that $\vec{\beta}_1, \vec{\beta}_2, \vec{\beta}_3$ is a basis for $\mathbb{R}^3$. Will the vectors $\vec{\beta}_1, \vec{\beta}_1 + \vec{\beta}_2, \vec{\beta}_1 + \vec{\beta}_2 + \vec{\beta}_3$ always be a basis for $\mathbb{R}^3$? (Prove your answer.)

6. Can a finite dimensional vector space have an infinite dimensional subspace? (Prove your answer.)

7. A *lower triangular* matrix is a square matrix in which all of the entries above the diagonal are zero. For example, here is an upper triangular 5×5 matrix:

$$\begin{pmatrix}3&0&0&0&0\\5&2&0&0&0\\1&7&8&0&0\\8&4&3&1&0\\5&0&2&3&7\end{pmatrix}$$

Let $T_{n\times n}$ be the set of lower triangular matrices of size n. T is a subspace of $M_{n\times n}$, the vector space of all $n \times n$ matrices. What is the dimension of $T_{n\times n}$? Describe a basis for $T_{n\times n}$. You do not have to prove your answer rigorously, but you should explain why your answer is true.