This homework is due on Monday, April 6.

1. Suppose that $h\colon \mathbb{R}^3 \to \mathbb{R}^2$ is a homomoprhism such that $h(\vec{e}_1) = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$, $h(\vec{e}_2) = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$, and $h(\vec{e}_3) = \begin{pmatrix} -3 \\ 1 \end{pmatrix}$. Find the matrix that represents h, using the standard bases for $\mathbb{R}^3$ and $\mathbb{R}^2$. (This problem is very short. The straightforward technique for finding a representation matrix using any bases was covered in class. The textbook is confusing on this.)

2. Let h be the homomorphism from problem **1**. Find the kernel of h. What is the nullity of h? What is the rank of h?

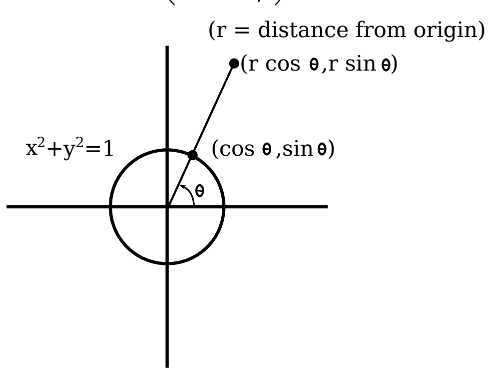
3. Consider the bases $B = \left\langle \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \right\rangle$ for$\mathbb{R}^3$ and $D = \left\langle \begin{pmatrix} 0 \\ 1 \end{pmatrix} \begin{pmatrix} -1 \\ 0 \end{pmatrix} \right\rangle$ for $\mathbb{R}^2$. Find the matrix $Rep_{B,D}(h)$, where h is still the homomorphism defined in problem **1**.

4. Let $\mathscr{P}_n$ be the vector space of polynomials of degree less than or equal to n. Recall that the derivative function, d mapping $\mathscr{P}_n$ to $\mathscr{P}_{n-1}$ by $d(p(x)) = p'(x)$ is a homomorphism. Find the matrix, $Rep_{B,D}(d)$ that represents $d\colon \mathscr{P}_4 \to \mathscr{P}_3$ with respect to the usual bases, $B = \langle 1, x, x^2, x^3 \rangle$ and $D = \langle 1, x, x^2 \rangle$. What is the representation of d mapping $\mathscr{P}_n$ to $\mathscr{P}_{n-1}$ using the usual bases?

5. Suppose that we use the basis $B = \langle 1, x, x^2, x^3 \rangle$ for $\mathscr{P}_4$, but use the basis $E = \langle x^2 - 3x, 2x+1, 2x-1 \rangle$ for $\mathscr{P}_3$. Find the representation matrix $Rep_{B,E}$ for the derivative function $d\colon \mathscr{P}_4 \to \mathscr{P}_3$.

6. Suppose that V and W are vector spaces and that $f\colon V \to W$ is a homomorphism. Suppose that $S \subseteq V$ and that S spans V. Show that the set $f(S)$ spans the range space of f, $\mathscr{R}(f)$. (Note: $\mathscr{R}(f) = \{f(\vec{v}) \mid \vec{v} \in V\}$, and $f(S) = \{f(\vec{v}) \mid \vec{v} \in S\}$.)

7. It was claimed in class that the matrix $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$ represents the linear transformation R_θ, which rotates a vector by θ degrees about the origin. Verify that this is true. Note that $(\cos\theta, \sin\theta)$ is the point on the unit circle at an angle of θ from the x-axis. This means that any vector in the plane can be represented as $\begin{pmatrix} r \cdot \cos\varphi \\ r \cdot \sin\varphi \end{pmatrix}$ as shown in this diagram:



So you only need to multiply $\begin{pmatrix} r \cdot \cos\varphi \\ r \cdot \sin\varphi \end{pmatrix}$ by the matrix and apply the angle sum formulas to the result. (Look up the angle sum formulas for sine and cosine if necessary!)