This homework is due on Monday, April 13.

1. Compute the matrix product $\begin{pmatrix} 2 & -1 \\ -3 & 1 \\ 2 & 4 \end{pmatrix} \begin{pmatrix} 1 & 0 & 3 \\ 2 & -4 & 5 \end{pmatrix}$.

2. Find the inverse of the matrix $\begin{pmatrix} -1 & 3 & 0 \\ 2 & -1 & 5 \\ 1 & 2 & -5 \end{pmatrix}$, using the row reduction method.

3. Show that a 2×2 matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ does **not** have an inverse if and only if one row of the matrix is a scalar multiple of the other row.

4. The $n \times n$ identity matrix, I_n, has the property that it is its own inverse. That is, the product $I_n I_n$ is equal to I_n.. There are other $n \times n$ matrices that have the same property; that is, $AA = I_n$.

a) Describe all **diagonal** $n \times n$ matrices D that have the property $DD = I_n$.

b) Let S be the 2×2 matrix $S = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$. Calculate the matrix product SS to see that S is its own inverse.

c) The matrix S from the previous part is a **permutation matrix**; multiplying a $2 \times m$ matrix on the left by S will swap the two rows of that matrix, so SS is the matrix that you get by swapping the rows of S, producing the identity matrix. More generally, what $n \times n$ permutation matrices, P, will have the property that P is its own inverse? (You might not find them all, but you should be able to specify some of them.)

d) Find a 3×3 permutation matrix A that has the property $AAA = I_3$.

5. The change of basis matrix for bases B and D of the same vector space V is defined to be $Rep_{B,D}(id)$, where $id\colon V \to V$ is the identity map.

a) Find the change of basis matrix, $Rep_{B,D}(id)$, for the bases $B = \langle x+1, x-2, x^2 \rangle$ and $D = \langle 2, x^2, -x \rangle$ of the vector space $\mathscr{P}_3$ of polynomials of degree less than or equal to 4.

b) Check that the representation of $1 - 2x + 3x^2$ in the basis B is $\begin{pmatrix} -1 \\ -1 \\ 3 \end{pmatrix}$.

c) Let A be the change of basis matrix $Rep_{B,D}(id)$ found in part a). Check that $A \cdot \begin{pmatrix} -1 \\ -1 \\ 3 \end{pmatrix}$ is in fact the representation of $1 - 2x + 3x^2$ in the basis D.