The first test for this course will be given in class on Friday, February 21. It covers all of the material that we have done in Chapters 1 and 2, up to Chapter 2, Section II.

For the test, you should know and understand all the definitions and theorems that we have covered. You should be able to work with matrices and systems of linear equations. The test will include some "short essay" questions that ask you to define something, or discuss something, or explain something, and so on. Other than that, you can expect most of the questions to be similar to problems that have been given on the homework. You can expect to do a few proofs, but they will be fairly straightforward.

Here are some terms and ideas that you should be familiar with for the test:

systems of linear equations

solution set of a linear system of equations

Gaussian elimination

the three row operations

notations for row operations: $k\rho_i + \rho_j$, $\rho_i \leftrightarrow \rho_j$, $k\rho_j$

row operations are reversible

applying row operations to a system of equations

echelon form for a system of linear equations

matrix; rows and columns of a matrix; $m \times n$ matrix

representing a system of linear equations as an augmented matrix

row operations and echelon form for matrices

applying row operations to a matrix

vectors in $\mathbb{R}^n$; column vectors and row vectors

vector addition and scalar multiplication for vectors in $\mathbb{R}^n$

linear combination of vectors in $\mathbb{R}^n$

expressing the solution set of a linear system in vector form

homogeneous system of linear equations

associated homogeneous system for a system of linear equations

a solution set has the form $\{\vec{p}+\vec{h} \,|\, \vec{h}$ solves the homogeneous system$\}$; $\vec{p}$ is a particular solution

solution set of a homogeneous system has the form $\{a_1\vec{v}_1 + a_2\vec{v}_2 + \cdots a_k\vec{v}_k \,|\, a_1, a_2, \dots, a_k \in \mathbb{R}\}$

a linear system of equations can have zero, one, or an infinite number of solutions

a homogeneous system can have one solution (namely $\vec{0}$) or an infinite number of solutions

leading variables in a system in echelon form

free variables

singular and non-singular matrices

dot product of vectors in $\mathbb{R}^n$: $\vec{u} \cdot \vec{v} = v_1 u_1 + v_2 u_2 + \cdots + v_n u_n$

length of a vector in $\mathbb{R}^n$: $|\vec{v}| = \sqrt{\vec{v} \cdot \vec{v}} = \sqrt{v_1^2 + v_2^2 + \cdots + v_n^2}$

angle between two vectors in $\mathbb{R}^n$: $\cos(\theta) = \frac{\vec{v} \cdot \vec{u}}{|\vec{v}||\vec{u}|}$

orthogonal vectors in $\mathbb{R}^n$, $\vec{v} \cdot \vec{u} = 0$

reduced row echelon form; Gauss-Jordan reduction

row equivalence of matrices

every matrix is row equivalent to a unique reduced row echelon form matrix

a square matrix is non-singular iff it is row equivalent to the identity matrix

linear combination lemma: a linear combination of linear combinations is a linear combination

vector space: closure, commutativity, associativity, zero vector, additive inverse, etc.

$\mathbb{R}^n$ as a vector space with the usual vector addition and scalar multiplication

the set of polynomials, $\mathscr{P}_d = \{p(x) \,|\, p \text{ is a polynomial of degree } \le d\}$, as a vector space

the set of all polynomials, $\mathscr{P}$, as a vector space

the set $\mathscr{C}$ of continuous functions from $\mathbb{R}$ to $\mathbb{R}$, as a vector space

showing that a set with addition and scalar multiplication operations is or is not a vector space

subspace of a vector space

proving a subset is a subspace: non-empty and closed under addition and scalar multiplication

the solution set of a homogeneous system of linear equations in n variables is a subspace of $\mathbb{R}^n$

the span of a subset of vector space (the set of linear combinations of elements of the subset)

a span is a subspace

linearly dependent subsets and linearly independent subset of a vector space

$\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_k$ are linearly independent iff $c_1\vec{v}_1 + c_2\vec{v}_2 + \cdots + c_k\vec{v}_k = \vec{0}$ implies $c_1 = c_2 = \cdots = c_k = 0$

the vectors $\vec{e}_1, \ldots, \vec{e}_n$ in $\mathbb{R}^n$ are linearly independent and span $\mathbb{R}^n$

the non-zero rows of an echelon form matrix are linearly independent

using linear equations to test spanning, linear independence, etc.