The exam will be given in our regular classroom at the officially designated time, 8:30 AM on Sunday, December 15. There will be some emphasis on material covered since the second in-class test, but you are responsible for all of the material covered in the textbook, that is, Chapters 1 through 4. You are not responsible for any of the material on metric spaces. You might want to review the study guides from the tests and my sample solutions to homework exercises, all of which are available on the course web page. The exam will be about 6 pages long and should not take the entire three hour exam period. However, you can use the three hours if you need them.s

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Here is a list of some major topics from Chapters 1 through 3:

definition of the real numbers as Dedekind cuts of the rational numbers.

upper and lower bounds; least upper bounds and greatest lower bounds.

Least Upper Bound Property of the real numbers.

Archimedian Property of the real numbers.

open cover of a set of real numbers.

subcover of an open cover.

the Heine-Borel Theorem.

accumulation point of a set (also known as cluster point).

Bolzano-Weierstrass Theorem.

$\lim_{n\to a} f(x)$ and $\lim_{n\to\infty} f(x)$.

continuity of a function $f(x)$ at a point $x = a$.

Intermediate Value Theorem.

uniformly continuous function on an interval.

every continuous function on a closed bounded interval is unifomly continuous.

Extreme Value Theorem.

the definition of the derivative as a limit.

basic laws of differentiation (sum rule, product rule, chain rule, etc.).

the Dirichlet function $D(x)$

Mean Value Theorem.

partition of an interval $[a, b]$.

upper sums and lower sums, $U(P, f)$ and $L(P, f)$.

integrable function on $[a, b]$; definition of the integral in terms of upper and lower sums.

if f is continuous on $[a, b]$, then it is integrable on $[a, b]$.

if f is increasing or decreasing on $[a, b]$, then it is integrable on $[a, b]$.

First and Second Fundamental Theorems of Calculus.

Here is a list of some of the things that you should know from Chapter 4:

infinite sequences of numbers and the notation $\{a_n\}_{n=1}^{\infty}$.

convergence of a sequence: $\lim_{n\to\infty} a_n = L$ if for every $\epsilon > 0$, there is an integer N such that for all $n \geq N$, $|a_n - L| < \epsilon$.

bounded sequence.

Theorem: Every convergent sequence is bounded.

Theorem: If $\lim_{n\to\infty} a_n = L$ and $\lim_{n\to\infty} b_n = M$, then $\lim_{n\to\infty}(a_n + b_n) = L + M$ — and similar results for subtraction, multiplication, and division.

monotone, increasing, and decreasing sequences.

Theorem: Monotone Convergence Theorem: A bounded monotone sequence is convergent.

Cauchy sequence: $\{a_n\}_{n=1}^{\infty}$ is Cauchy if for every $\epsilon > 0$, there is an integer N such that for all $n \geq N$ and $m \geq N$, $|a_n - a_m| < \epsilon$.

Theorem: A sequence is convergent if and only if it is Cauchy.

infinite series of numbers and the notation $\sum_{n=1}^{\infty} a_n$.

partial sums of a series.

convergence of a series, defined as the convergence of the sequence of partial sums.

the harmonic series, $\sum_{n=1}^{\infty} \frac{1}{n}$

p-series, $\sum_{n=1}^{\infty} \frac{1}{n^p}$, convergent if and only if $p > 1$.

geometric series, $\sum_{n=0}^{\infty} r^n$ (convergent to $\frac{1}{1-r}$ if $|r| < 1$ and divergent otherwise).

Theorem: If $\sum_{n=1}^{\infty} a_n = L$ and $\sum_{n=1}^{\infty} b_n = M$, then $\sum_{n=1}^{\infty}(a_n + b_n) = L + M$ — and similar results for constant multiples and subtraction (but not for mutiplication or division).

absolute convergence.

Theorem: If a series converges absolutely, then it converges.

Theorem: The n-th term test: If $\sum_{n=1}^{\infty} a_n$ converges, then $\lim_{n\to\infty} a_n = 0$. Equivalently, if $\lim_{n\to\infty} a_n \neq 0$, then $\sum_{n=1}^{\infty} a_n$ diverges.

Theorem: The Comparison Test: Suppose $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are series of positive terms. If the series $\sum_{n=1}^{\infty} b_n$ converges, and if there is an integer N such that for all $n \geq N$, $a_n \leq b_n$, then the series $\sum_{n=1}^{\infty} a_n$ converges. If the series $\sum_{n=1}^{\infty} b_n$ diverges, and if there is an integer N such that for all $n \geq N$, $a_n \geq b_n$, then the series $\sum_{n=1}^{\infty} a_n$ diverges.

Theorem: The Ratio Test: If $\lim_{n\to\infty} \frac{|a_{n+1}|}{|a_n|}$ exists, then if the limit is less than 1 (including 0), the series $\sum_{n=1}^{\infty} a_n$ converges absolutely, and if the limit is greater than 1 (including ∞) then the series diverges.

Theorem: The Root Test: If $\lim_{n\to\infty} \sqrt[n]{|a_n|}$ exists, then if the limit is less than 1 (including 0), the series $\sum_{n=1}^{\infty} a_n$ converges absolutely, and if the limits is greater than 1 (including ∞) then the series diverges.

conditional convergence.

alternating series.

Theorem: Alternating Series Test: If the sequence of non-negative terms $\{a_n\}_{n=1}^{\infty}$ is decreasing and converges to zero, then the series $\sum_{n=1}^{\infty}(-1)^n a_n$ is convergent sequences of functions, $\{f_n\}_{n=1}^{\infty}$.

pointwise convergence of a sequence of functions on a domain D.

uniform convergence of a sequence of functions on a domain D: $\{f_n\}_{n=1}^{\infty}$ converges uniformly to f on D if for every $\epsilon > 0$, there is an integer N such that for all $n \geq N$ and all $x \in D$, $|f_n(x) - f(x)| < \epsilon$.

Theorem: The uniform limit of a sequence of continuous functions is continuous.

Theorem: If $\{f_n\}_{n=1}^{\infty}$ converges uniformly on the interval $[a, b]$, then the limit function is integrable on $[a, b]$ and $\displaystyle\int_a^b \left(\lim_{n\to\infty} f_n(x)\right) dx = \lim_{n\to\infty} \left(\int_a^b f_n(x)\, dx\right)$.

Theorem: If $\{f_n\}_{n=1}^{\infty}$ converges (pointwise) on an interval I, and if each f_n is differentiable on I, and if each f_n is continuous, and if $\{f_n'\}_{n=1}^{\infty}$ converges uniformly on I, then $\displaystyle\lim_{n\to\infty} f_n$ is differentiable on I, and $\displaystyle\frac{d}{dx}\left(\lim_{n\to\infty} f_n(x)\right) = \lim_{n\to\infty} f_n'(x)$, for $x \in I$.

series of functions, $\displaystyle\sum_{n=1}^{\infty} f_n(x)$.

convergence of a series of functions.

uniform convergence of a series of functions and its consequences.

power series, $\displaystyle\sum_{n=0}^{\infty} a_n(x-a)^n$.

radius of convergence of a power series.

interval of convergence of a power series.

term-by-term differentiation and integration of power series.

Taylor series of a function, $\displaystyle\sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$.

the Taylor series of an infinitely differentiably function does not necessarily converge to that function.

real-analytic function.