This homework covers the compactness handout, Section 4.1, and Section 2.2. It is due on Friday, September 20. Because there is a test coming up on Wednesday of next week, I would like to hand out my answers for this homework when I collect the homework on Friday. That means that there won't be time to do rewrites on this homework. If you are concerned that your answers might not be correct, you should bring your work in on Thursday or before class on Friday to go over it with me.

Note that for now, sequences are sequences of real numbers, and functions are functions from $\mathbb{R}$ to $\mathbb{R}$. Later, we will look at sequences and functions in general metric spaces, but not until after the test.

Exercises 1 to 5: The first five exercises on this homework are the first five exercises on the compactness handout. (The sixth exercise from that handout is not assigned.)

Exercise 6: Prove that the limits of a sequence, $\{x_n\}_{n=1}^{\infty}$, is unique. That is, prove that if the sequence converges to L_1 and also converges to L_2, then $L_1 = L_2$. (See the proof of Theorem 2.3.1 on page 65 of the textbook, which proves the corresponding result for limits of functions.)

Exercise 7: Prove Theorem 1.4.5, Part b, using the definition of limit: Let $\{x_n\}_{n=1}^{\infty}$ be a convergent sequence of real numbers, and let $c \in \mathbb{R}$. Then $\lim\limits_{n\to\infty}(c \cdot x_n) = c \cdot \lim\limits_{x\to\infty} x_n$.

Exercise 8: Do Exercise 4.1.3 from the textbook: For $n \geq 1$, define

$$a_n = 1 + \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \cdots + \frac{1}{2^n}$$

Show that $a_n = 2 - \frac{1}{2^n}$, and then use that fact to show that $\{a_n\}_{n=1}^{\infty}$ converges to 2.

Exercise 9: Use the definition of limit of function to prove directly that $\lim\limits_{x\to 3}(2x-1) = 5$