Due ***in class*** *on Wednesday, April 19*

1. Compute both the 1-level and the 2-level two-dimensional Haar wavelet transform of the following 4×4 matrix:

$$\begin{pmatrix} 2 & -2 & 0 & 0 \\ -2 & -2 & 0 & 0 \\ 3 & 1 & 4 & 4 \\ 5 & 1 & 0 & 2 \end{pmatrix}$$

2. A multiresolution analysis based on the 2-level Haar transforms of the matrix in problem **1** would write that matrix as the sum of seven matrixes, $A^2 + (H^2 + V^2 + D^2) + (H^1 + V^1 + D^1)$. Write out this multiresolution analysis explicitly as the sum of seven 4×4 matrices.

3. Let $\{\mathbf{b}_1, \mathbf{b}_2, \dots, \mathbf{b}_n\}$ be an orthonormal basis of $\mathbb{R}^n$. Write $\mathbf{b}_i = (b^i_1, b^i_2, \dots, b^i_n)$, for $i = 1, 2, \dots, n$. We can associate this basis with the matrix

$$B = \begin{pmatrix} b^1_1 & b^1_2 & \dots & b^1_n \\ b^2_1 & b^2_2 & \dots & b^2_n \\ \vdots & \vdots & & \vdots \\ b^n_1 & b^n_2 & \dots & b^n_n \end{pmatrix}$$

This matrix is the matrix of the linear transformation A from $\mathbb{R}^n$ to $\mathbb{R}^n$ that maps $\mathbf{b}_i$ to the standard basis vector $\mathbf{e}_i$, for $i = 1, 2, \dots, n$. For $\mathbf{x} \in \mathbb{R}^n$, the components of $A(\mathbf{x})$ give the expansion of $\mathbf{x}$ in the basis $\{\mathbf{b}_1, \mathbf{b}_2, \dots, \mathbf{b}_n\}$.

Now, we also have the orthonormal basis $\{\mathbf{b}_i \otimes \mathbf{b}_j \,|\, 1 \le i \le n,\ 1 \le j \le n\}$ of the vector space $\mathbb{R}^n \otimes \mathbb{R}^n$ and a corresponding linear transformation from $\mathbb{R}^n \otimes \mathbb{R}^n$ to itself. The effect of this transformation on an $n \times n$ matrix M can be computed by applying A to the rows of M and then to the columns of the resulting matrix (or *vice* versa). Show that this value can also be computed as the matrix product BMB^T. [Note: This is not difficult. It's just a matter of looking at how the matrix product is computed and keeping the definitions straight.]

4. The two-dimensional Discrete Fourier Transform from $\mathbb{C}^n \otimes \mathbb{C}^n$ to $\mathbb{C}^n \otimes \mathbb{C}^n$ can be defined in terms of the basis $\{w_k \otimes w_\ell \,|\, 0 \le k < n,\ 0 \le \ell < n\}$. In this definition, the entries in the DFT of an $n \times n$ matrix M are given by the inner products $\langle M, w_k \otimes w_\ell \rangle$.

a) Find the number of operations needed to compute the two-dimensional DFT of M using this definition, giving your answer in the form "*some constant times* $f(n)$" for a function $f(n)$.

b) Suppose that the DFT of M is computed by applying the one-dimensional DFT to the rows of M and then to the columns of the resulting matrix. Find the number of operations in this case.

c) Suppose that n is a power of 2, so that the FFT can be used to compute the DFT. When the FFT is used in part **b)**, what is the number of operations?

d) Suppose that you want to compute the two-dimensional DFT of a 1024-by-1024 pixel image. ($1024 = 2^{10}$.) Write a paragraph comparing the computational cost of the methods in parts **a)**, **b)**, and **c)**.

5. State the topic of your final project, write a short description of what it will be about, and list several references that you will use in the project.